

SHORT NOTES ON MATHEMATICS

Every chapter of Class 12 in NCERT order, then the Class 11 topics that JEE needs — key results, shortcuts and 485 solved examples

Relations and Functions · Inverse Trigonometric Functions · Matrices · Determinants · Continuity and Differentiability · Application of Derivatives · Integrals · Application of Integrals · Differential Equations · Vector Algebra · Three Dimensional Geometry · Linear Programming · Probability · Trigonometric Functions · Complex Numbers · Quadratic Equations · Permutations and Combinations · Binomial Theorem · Sequences and Series · Conic Sections · Limits · Statistics

Short Notes on Mathematics

for Class 12, JEE Main and JEE Advanced

MTA Team

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Numerical answers to the solved examples have been checked a second time by computer; the book is not printed if any of these checks fails. If you still find a mistake, please write to us.

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How to use these notes

These are revision notes: short, exact and complete. They are not a first textbook. Read a chapter of NCERT first; come here to fix every result in memory and to see how each one is used in a JEE question.

Order. Part I follows the chapters of the NCERT Class 12 textbook, in the same order. Part II then gives, in NCERT Class 11 order, the topics of Class 11 that JEE Main and JEE Advanced keep asking about.

In every chapter:

- boxed — key definitions and results, to be learnt by heart;
- red — shortcuts, traps and common mistakes;
- **Solved examples** — questions of the JEE type, each with a complete solution. Cover the solution, try the question, then check.

In the last week before the examination, reading only the boxes of every chapter is a complete revision of the syllabus.

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PART I

Class 12

Relations and Functions

A relation pairs elements of two sets; a function is a relation in which every input has exactly one output. JEE asks about the type of a relation, counting functions of each kind, and composition and inverse.

1.1 Relations

Definitions

A relation R from A to B is a subset of $A \times B$. If $n(A) = p$ and $n(B) = q$, there are 2^{pq} relations from A to B . On a set A , a relation R is

- **reflexive** if $(a, a) \in R$ for every $a \in A$;
- **symmetric** if $(a, b) \in R \Rightarrow (b, a) \in R$;
- **transitive** if $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$;
- an **equivalence relation** if it is all three. It splits A into disjoint *equivalence classes*.

Counting relations on a set with n elements

all relations	2^{n^2}
reflexive	2^{n^2-n}
symmetric	$2^{n(n+1)/2}$
reflexive and symmetric	$2^{n(n-1)/2}$
antisymmetric	$2^n \cdot 3^{n(n-1)/2}$

Remember

The empty relation on a non-empty set is symmetric and transitive but not reflexive. The identity relation $\{(a, a)\}$ is the smallest equivalence relation, $A \times A$ the largest.

Example 1.1 On $\mathbb{Z}$, $R = \{(a, b) : a - b \text{ is divisible by } 5\}$. Show that R is an equivalence relation and find its classes.

Solution. $a - a = 0$ is divisible by 5 (reflexive); if $5 \mid a - b$ then $5 \mid b - a$ (symmetric); if $5 \mid a - b$ and $5 \mid b - c$ then $5 \mid a - c$ (transitive). The classes are the five residue classes $[0], [1], [2], [3], [4]$: numbers leaving the same remainder on division by 5.

Answer: an equivalence relation with 5 classes

Example 1.2 Let $A = \{1, 2, 3\}$. How many relations on A containing $(1, 2)$ and $(2, 3)$ are reflexive and transitive but not symmetric?

Solution. Reflexive: all (a, a) are in. Transitive with $(1, 2), (2, 3)$: $(1, 3)$ must be in. This gives

$$R_0 = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\},$$

which is transitive and not symmetric. Adding any one of $(2, 1), (3, 2), (3, 1)$ forces, by transitivity, further pairs: adding $(2, 1)$ gives a transitive relation $R_0 \cup \{(2, 1)\}$; adding $(3, 2)$ gives $R_0 \cup \{(3, 2)\}$; adding $(3, 1)$ forces $(3, 2)$ and $(2, 1)$, i.e. the whole of $A \times A$, which is symmetric. Adding two of them also forces all of $A \times A$. So the relations are $R_0, R_0 \cup \{(2, 1)\}$ and $R_0 \cup \{(3, 2)\}$.

Answer: 3

1.2 Functions

Types of function $f : A \rightarrow B$, with $n(A) = m, n(B) = n$

- **one-one (injective):** $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. Number of one-one functions: ${}^nP_m = \frac{n!}{(n-m)!} (m \leq n)$.
- **onto (surjective):** range = B . Number of onto functions ($m \geq n$):

$$\sum_{r=0}^n (-1)^r \binom{n}{r} (n-r)^m.$$

- **bijective:** one-one and onto; possible only if $m = n$; there are $n!$ of them.
- All functions: n^m .

Testing one-one and onto

A continuous function on an interval is one-one if and only if it is strictly monotonic: check the sign of $f'(x)$. For onto, compare the range with the co-domain. A horizontal line meeting the graph twice shows that f is not one-one.

Example 1.3 Find the number of onto functions from $\{1, 2, 3, 4, 5\}$ to $\{a, b, c\}$.

Solution. $3^5 - \binom{3}{1}2^5 + \binom{3}{2}1^5 = 243 - 96 + 3 = 150$.

Answer: 150

Example 1.4 Is $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 + 3x + 5$ a bijection?

Solution. $f'(x) = 3x^2 + 3 > 0$: f is strictly increasing, hence one-one. It is continuous with $f(x) \rightarrow \pm\infty$ as $x \rightarrow \pm\infty$, so its range is $\mathbb{R}$: onto.

Answer: yes

1.3 Composition and inverse

Composition

$(g \circ f)(x) = g(f(x))$, defined when the range of f lies in the domain of g .

- In general $g \circ f \neq f \circ g$, but composition is associative: $h \circ (g \circ f) = (h \circ g) \circ f$.
- If f and g are one-one (onto), so is $g \circ f$.
- If $g \circ f$ is one-one then f is one-one; if $g \circ f$ is onto then g is onto.

Inverse

f has an inverse if and only if it is a bijection. Then $f^{-1}(y) = x \iff f(x) = y$, and

$$f \circ f^{-1} = I_B, \quad f^{-1} \circ f = I_A, \quad (g \circ f)^{-1} = f^{-1} \circ g^{-1}.$$

The graphs of f and f^{-1} are mirror images in the line $y = x$; where they meet on a monotonic increasing f , they meet on $y = x$.

Example 1.5 $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 3x - 2$ and $g(x) = x^2 + 1$. Find $(g \circ f)(x)$, $(f \circ g)(x)$ and $f^{-1}(x)$.

Solution. $(g \circ f)(x) = (3x - 2)^2 + 1 = 9x^2 - 12x + 5$; $(f \circ g)(x) = 3(x^2 + 1) - 2 = 3x^2 + 1$. From $y = 3x - 2$:
 $f^{-1}(x) = \frac{x+2}{3}$.

Answer: $9x^2 - 12x + 5$; $3x^2 + 1$; $(x + 2)/3$

Example 1.6 Let $f : [\frac{1}{2}, \infty) \rightarrow [\frac{3}{4}, \infty)$, $f(x) = x^2 - x + 1$. Solve $f(x) = f^{-1}(x)$.

Solution. f is increasing on $[\frac{1}{2}, \infty)$, so f and f^{-1} can only meet on the line $y = x$: $x^2 - x + 1 = x$, i.e. $(x - 1)^2 = 0$, $x = 1$.

Answer: $x = 1$