

SHORT NOTES ON PHYSICS



Every chapter of Class 12 in NCERT order, then the Class 11 chapters that JEE needs — key results, shortcuts and 523 solved examples

Electric Charges and Fields · Electrostatic Potential and Capacitance · Current Electricity · Moving Charges and Magnetism · Magnetism and Matter · Electromagnetic Induction · Alternating Current · Electromagnetic Waves · Ray Optics and Optical Instruments · Wave Optics · Dual Nature of Radiation and Matter · Atoms · Nuclei · Semiconductor Electronics · Units and Measurements · Motion in a Straight Line and in a Plane · Laws of Motion · Work, Energy and Power · System of Particles and Rotational Motion · Gravitation · Mechanical Properties of Solids · Mechanical Properties of Fluids · Thermal Properties of Matter · Thermodynamics · Kinetic Theory · Oscillations · Waves

Short Notes on Physics

for Class 12, JEE Main and JEE Advanced

MTA Team

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Numerical answers to the solved examples have been checked a second time by computer; the book is not printed if any of these checks fails. If you still find a mistake, please write to us.

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How to use these notes

These are revision notes: short, exact and complete. They are not a first textbook. Read a chapter of NCERT first; come here to fix every result in memory and to see how each one is used in a JEE question.

Order. Part I follows the chapters of the NCERT Class 12 textbook, in the same order. Part II then gives, in NCERT Class 11 order, the chapters of Class 11 that JEE Main and JEE Advanced also test.

In every chapter:

- boxed — key definitions and results, to be learnt by heart;
- red — shortcuts, traps and common mistakes;
- **Solved examples** — questions of the JEE type, each with a complete solution. Cover the solution, try the question, then check.

In the last week before the examination, reading only the boxes of every chapter is a complete revision of the syllabus.

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PART I

Class 12

Electric Charges and Fields

Charges at rest exert forces given by Coulomb's law, and these forces add as vectors. The electric field describes the force per unit charge at every point; for symmetric charge distributions Gauss's law gives it in one line.

1.1 Electric charge

Basic facts

- Charge is a scalar; SI unit coulomb (C). $e = 1.6 \times 10^{-19}$ C; electron $-e$, proton $+e$, neutron 0.
- Practical unit (battery rating): $1 \text{ mAh} = 10^{-3} \text{ A} \times 3600 \text{ s} = 3.6 \text{ C}$.
- $m_p/m_e \approx 1836$. Electrons, being light and loosely bound in outer shells, are the particles transferred when bodies are charged; protons and neutrons stay in the nucleus.

Properties of charge

1. **Quantisation:** $q = ne, n \in \mathbb{Z}$. A body can gain or lose only whole electrons.
2. **Charge needs mass:** charge is always carried by a particle with mass; no mass, no charge (a photon has neither).
3. **Conservation:** the total charge of an isolated system is constant; charge is neither created nor destroyed, only transferred (also in pair production and annihilation).
4. **Additivity:** total charge is the algebraic sum of the charges.
5. **Invariance:** charge does not depend on the speed of the body (or the frame of reference). Mass does: $m = m_0 / \sqrt{1 - v^2/c^2}$.

Methods of charging

- **Friction:** rubbing transfers electrons, the two bodies get equal and opposite charges (glass rod rubbed with silk: rod $+$, silk $-$). It works only if at least one body is an insulator.
- **Conduction** (contact): if two identical conductors touch, the total charge is shared equally. $+10 \mu\text{C}$ and $0 \rightarrow +5 \mu\text{C}$ each; q_1 and $q_2 \rightarrow \frac{q_1+q_2}{2}$ each (with signs).
- **Induction:** a charged body brought near a conductor separates its charges (opposite charge on the near side). The attraction to the near charges is stronger than the repulsion from the far ones, so a charged body attracts a neutral one.

Test of electrification

Attraction does not prove that a body is charged (a charged body also attracts a neutral one by induction). **Repulsion** is the sure test: only two charged bodies (with like charges) repel.

Example 1.1 A mobile phone battery is rated 4000 mAh. How much charge does it deliver when fully used, and how many electrons is that?

Solution. $Q = 4000 \times 3.6 = 14400 \text{ C}$, $n = Q/e = 14400 / (1.6 \times 10^{-19}) = 9 \times 10^{22}$.

Answer: $1.44 \times 10^4 \text{ C}$, 9×10^{22} electrons

1.2 Coulomb's law

Coulomb's law

Two point charges q_1, q_2 at rest, a distance r apart in vacuum (air):

$$F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} = \frac{k q_1 q_2}{r^2}, \quad k = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}, \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}.$$

The forces are equal and opposite (Newton's third law holds) and act along the line joining the charges; like charges repel, unlike attract. **Vector form** (force on q_2 due to q_1):

$$\vec{F}_{21} = \frac{k q_1 q_2}{r^2} \hat{r}_{12} = \frac{k q_1 q_2}{|\vec{r}_2 - \vec{r}_1|^3} (\vec{r}_2 - \vec{r}_1).$$

Write the magnitude first, then multiply by the unit vector $\vec{AB}/|\vec{AB}|$ in the direction of the force.

In a medium

In a medium of relative permittivity (dielectric constant) $\epsilon_r = K$, $\epsilon_m = \epsilon_0 \epsilon_r$ and

$$F_m = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2} = \frac{F_{\text{air}}}{\epsilon_r}, \quad F_{\text{air}} - F_m = \frac{k q_1 q_2}{r^2} \left(1 - \frac{1}{\epsilon_r}\right).$$

$\epsilon_r = 1$ for vacuum, ≈ 1.0006 for air, ≈ 81 for water, ∞ for a metal.

Medium: what really changes

The force that one charge exerts on the other (the Coulomb force between them) is the same in any medium. The medium is polarised, and the induced charges of the medium near each charge reduce the *net* force to F/ϵ_r .

Limitations of Coulomb's law

It holds exactly only for point charges (or spherically symmetric distributions, measured from the centres) at rest. For extended bodies integrate, and for moving charges magnetic forces also appear.

Superposition

The force on a charge is the vector sum of the forces due to each other charge, each computed as if the others were absent: $\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \dots$. For two forces of equal magnitude F with angle θ between them, $F_{\text{net}} = 2F \cos \frac{\theta}{2}$ along the bisector.

Example 1.2 A charge $q_1 = 2 \mu\text{C}$ is at $A(1 \text{ m}, 2 \text{ m})$ and $q_2 = 5 \mu\text{C}$ at $B(4 \text{ m}, 6 \text{ m})$. Find the force on q_2 in vector form.

Solution. $\vec{AB} = 3\hat{i} + 4\hat{j}$, $|\vec{AB}| = 5 \text{ m}$. $|\vec{F}| = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 5 \times 10^{-6}}{25} = 3.6 \times 10^{-3} \text{ N}$, directed from A to B (repulsion): $\vec{F} = 3.6 \times 10^{-3} \frac{3\hat{i} + 4\hat{j}}{5}$.

Answer: $\vec{F} = (2.16\hat{i} + 2.88\hat{j}) \times 10^{-3} \text{ N}$

Example 1.3 Charges of $2 \mu\text{C}$ and $3 \mu\text{C}$ are 10 cm apart in a medium of dielectric constant 4 . Find the net force on each and the reduction from its value in air.

Solution. $F_{\text{air}} = \frac{9 \times 10^9 \times 6 \times 10^{-12}}{0.01} = 5.4 \text{ N}$, $F_m = 5.4/4 = 1.35 \text{ N}$, reduction $5.4(1 - \frac{1}{4}) = 4.05 \text{ N}$.

Answer: 1.35 N (repulsive); reduced by 4.05 N

Example 1.4 A charge Q is divided into two parts q and $Q - q$, placed a fixed distance apart. For what q is the force between them maximum?

Solution. $F = kq(Q - q)/r^2$. $\frac{dF}{dq} = \frac{k}{r^2}(Q - 2q) = 0 \Rightarrow q = Q/2$ (and $d^2F/dq^2 < 0$, a maximum).

Answer: $q = Q/2$ (equal halves)

Example 1.5 Charges $+q$ and $+4q$ are fixed a distance l apart. Where must a third charge Q be placed so that it is in equilibrium, and what must Q be so that the whole system is in equilibrium?

Solution. Between the charges, at distance x from $+q$: $\frac{kqQ}{x^2} = \frac{k4qQ}{(l-x)^2} \Rightarrow l-x = 2x$, $x = l/3$ (for any Q). For $+q$ to be in equilibrium as well: $\frac{kq|Q|}{(l/3)^2} = \frac{kq \cdot 4q}{l^2} \Rightarrow |Q| = \frac{4q}{9}$, and Q must attract $+q$ towards $+4q$'s side, so Q is negative. ($+4q$ is then in equilibrium automatically.)

Answer: $l/3$ from $+q$ on the line between them; $Q = -\frac{4q}{9}$

1.3 Hanging charges and the image method

Two charged balls on threads

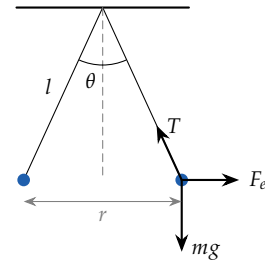
No gravity: the threads (length L each, from a common point) become one straight line, $\theta = \pi$, separation $2L$ (maximum) and tension $T = \frac{kq_1q_2}{(2L)^2}$.

With gravity: the forces on each ball are mg , T and F_e . If θ is the angle of a thread with the vertical,

$$T \cos \theta = mg, \quad T \sin \theta = F_e, \quad \tan \theta = \frac{F_e}{mg}.$$

The two balls make equal angles only if their masses are equal (the charges need not be equal: the forces on the two are equal anyway). Small angles ($\tan \theta \approx \sin \theta = r/2l$) with equal masses:

$$r^3 = \frac{q_1q_2 l}{2\pi\epsilon_0 mg}, \quad \text{so } \frac{r^3}{q_1q_2} = \text{const.}$$



Immersed in a liquid

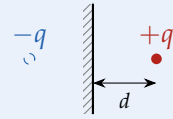
If the balls (density ρ_B) hang in a liquid of density ρ_L and dielectric constant K , buoyancy makes $g_{\text{eff}} = g(1 - \frac{\rho_L}{\rho_B})$ and the electric force becomes F_e/K . The angle is unchanged if

$$K = \frac{\rho_B}{\rho_B - \rho_L}.$$

Image method

A charge $+q$ at distance d from a large earthed conducting plane induces charge on the plane; outside the conductor its effect is exactly that of an image charge $-q$ at distance d behind the plane (a mirror image). So the plane attracts the charge with

$$F = \frac{kq^2}{(2d)^2}.$$



Example 1.6 Two identical balls, each of mass 10 g and carrying equal charges, hang from a common point on threads of length 1 m. In equilibrium each thread makes 30° with the vertical. Find the charge on each ball ($g = 10 \text{ m s}^{-2}$).

Solution. $r = 2l \sin 30^\circ = 1 \text{ m}$; $F_e = mg \tan 30^\circ = 0.1 \times 0.577 = 0.0577 \text{ N}$; $q = r\sqrt{F_e/k} = \sqrt{0.0577/(9 \times 10^9)} = 2.53 \times 10^{-6} \text{ C}$.

Answer: $q \approx 2.5 \mu\text{C}$

Example 1.7 The balls of the previous example are made of a material of density 1600 kg m^{-3} and the system is immersed in an oil of density 800 kg m^{-3} . The angle between the threads does not change. Find the dielectric constant of the oil.

Solution. Same angle: $\frac{F_e/K}{mg(1 - \rho_L/\rho_B)} = \frac{F_e}{mg} \Rightarrow K = \frac{\rho_B}{\rho_B - \rho_L} = \frac{1600}{800} = 2$.

Answer: $K = 2$

Example 1.8 Two equal small charged balls hang from a common point and are a distance r apart (small angles). The charge on each ball is doubled. Find the new separation.

Solution. $r^3 \propto q^2$, so $r'^3 = 4r^3$ and $r' = 4^{1/3}r = 2^{2/3}r \approx 1.59r$.

Answer: $2^{2/3}r \approx 1.59r$

1.4 Electric field

Definition

$$\vec{E} = \lim_{q_0 \rightarrow 0} \frac{\vec{F}}{q_0}, \quad \text{point charge: } E = \frac{kq}{r^2} \text{ (radially outward for } q > 0, \text{ inward for } q < 0).$$

Unit $\text{N C}^{-1} = \text{V m}^{-1}$. $\vec{E}$ is along the force on a positive test charge. Fields add as vectors (superposition). A charge in a field feels $\vec{F} = q\vec{E}$: along $\vec{E}$ for $q > 0$, opposite for $q < 0$.

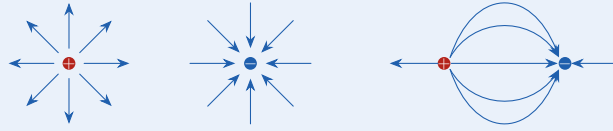
Why the test charge must be tiny

A finite test charge would push the source charges (on a conductor) and change the very field being measured. So $\vec{E} = \vec{F}/q_0$ only in the limit $q_0 \rightarrow 0$; the test charge is an infinitesimally small positive charge.

Field lines

- They start on positive charges and end on negative charges (or at infinity); the tangent at a point gives the direction of $\vec{E}$.
- Two field lines never cross (the field has one direction at each point). They do not form closed loops (electrostatic field is conservative).

- Their density shows the strength of the field; equal spacing means a uniform field.
- They meet a conductor's surface at right angles, and there are none inside a conductor in electrostatics.



Charged particle in a uniform field

At rest it accelerates along $\vec{F} = q\vec{E}$ with $a = qE/m$. A mass m with charge q moving under gravity in a uniform field moves as a projectile with an effective gravity

$$\vec{g}_{\text{eff}} = \vec{g} + \frac{q\vec{E}}{m} : \quad g_{\text{eff}} = g + \frac{qE}{m} (\vec{E} \text{ down}), \quad g_{\text{eff}} = g - \frac{qE}{m} (\vec{E} \text{ up}), \quad q > 0.$$

All projectile and pendulum formulas hold with $g \rightarrow g_{\text{eff}}$, e.g. $T = 2\pi\sqrt{l/g_{\text{eff}}}$; in a liquid also multiply g by $(1 - \rho_L/\rho_B)$: $g_{\text{eff}} = g(1 - \frac{\rho_L}{\rho_B}) - \frac{qE}{m}$ for an upward force qE .

A particle entering a field E perpendicular to its velocity u moves on a parabola; after a length L of the field its deflection is $y = \frac{qEL^2}{2mu^2}$.

Example 1.9 A simple pendulum of length 1 m has a bob of mass 10 g with charge $+10 \mu\text{C}$. A uniform field of 5000 N C^{-1} acts vertically upwards. Find the period ($g = 10 \text{ m s}^{-2}$).

Solution. $qE/m = 10^{-5} \times 5000/0.01 = 5 \text{ m s}^{-2}$ upwards, so $g_{\text{eff}} = 10 - 5 = 5 \text{ m s}^{-2}$ and $T = 2\pi\sqrt{1/5} = 2.81 \text{ s}$.

Answer: 2.81 s

Example 1.10 An electron enters, with speed $2 \times 10^7 \text{ m s}^{-1}$, the region between two plates where the field is 1000 N C^{-1} perpendicular to its velocity. The plates are 5 cm long. Find the deflection at exit ($e/m = 1.76 \times 10^{11} \text{ C kg}^{-1}$).

Solution. $y = \frac{eEL^2}{2mu^2} = \frac{1.76 \times 10^{11} \times 1000 \times (0.05)^2}{2 \times (2 \times 10^7)^2} = 5.5 \times 10^{-4} \text{ m}$, towards the positive plate.

Answer: 0.55 mm

1.5 Fields of continuous charge distributions

Method

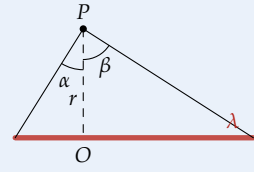
Take an element: $dq = \lambda dl$ (line), σdA (surface), ρdV (volume). Write $dE = k dq/r^2$, resolve into components, use symmetry to cancel some, and integrate the rest.

Straight rod, line charge density λ

At a point P at perpendicular distance r from the rod, whose ends make angles α and β with the perpendicular PO :

$$E_{\perp} = \frac{k\lambda}{r}(\sin \alpha + \sin \beta), \quad E_{\parallel} = \frac{k\lambda}{r}(\cos \alpha - \cos \beta),$$

$E_{\parallel}$ pointing away from the longer part ($\beta > \alpha$ in the figure). If both ends are on the same side of O (angles $\alpha < \beta$ measured on that side), $E_{\perp} = \frac{k\lambda}{r}(\sin \beta - \sin \alpha)$.



- Infinite rod: $E = \frac{2k\lambda}{r} = \frac{\lambda}{2\pi\epsilon_0 r}$, perpendicular to the rod.
- Semi-infinite rod, P opposite the end ($\alpha = 0, \beta = 90^\circ$): $E_{\perp} = E_{\parallel} = \frac{k\lambda}{r}$, so $\vec{E}$ is at 45° to the rod.
- Rod of length L , charge Q , on its perpendicular bisector: $E = \frac{2kQ}{r\sqrt{L^2 + 4r^2}}$.
- On the axis, at distance r from the near end: $E = \frac{kQ}{L} \left(\frac{1}{r} - \frac{1}{r+L} \right) = \frac{kQ}{r(r+L)}$.

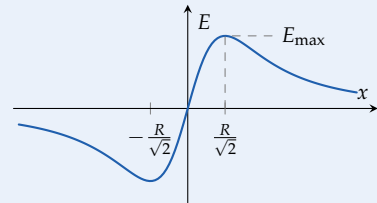
Ring and arc, radius R

Uniform ring of charge Q : at the centre $E_0 = 0$; on the axis at distance x ,

$$E = \frac{kQx}{(x^2 + R^2)^{3/2}} \text{ along the axis.}$$

Maximum at $x = \pm R/\sqrt{2}$:

$$E_{\max} = \frac{2kQ}{3\sqrt{3}R^2}.$$



For $x \gg R$, $E \approx kQ/x^2$ (a point charge).

- Ring with a small gap of width w ($\lambda = Q/2\pi R$): $E_0 = \frac{k\lambda w}{R^2}$, pointing towards the gap for $\lambda > 0$ (the field of the missing piece, reversed).
- Arc subtending angle ϕ at the centre: $E_0 = \frac{2k\lambda}{R} \sin \frac{\phi}{2}$ along the bisector; semicircle: $\frac{2k\lambda}{R}$.
- n equal charges q at the vertices of a regular polygon: on the axis, exactly as a ring of charge nq ; at the centre, $E = 0$.

Disc and sheets

- Uniform disc (σ , radius R), on the axis: $E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{x}{\sqrt{x^2 + R^2}} \right)$.
- Infinite thin sheet (non-conducting), $R \gg x$: $E = \frac{\sigma}{2\epsilon_0}$, the same at every distance.
- Charged conducting plate: no field inside; charge sits on both faces. With density σ on a face, the field just outside is $\frac{\sigma}{\epsilon_0}$. (A plate with total charge Q and face area A has $\sigma = Q/2A$ on each face, so $E = Q/2A\epsilon_0$ — the same as a thin sheet of Q .)
- Uniformly charged hemispherical shell (σ): at the centre of its base $E = \frac{\sigma}{4\epsilon_0}$; at every point of the plane base the field is perpendicular to the base (its magnitude grows towards the rim).

Example 1.11 A rod of length 20 cm carries a uniform charge of 4 nC. Find the field at a point on its axis 10 cm from one end.

Solution. $E = \frac{kQ}{r(r+L)} = \frac{9 \times 10^9 \times 4 \times 10^{-9}}{0.1 \times 0.3} = 1200 \text{ N C}^{-1}$, along the axis away from the rod.

Answer: $1.2 \times 10^3 \text{ N C}^{-1}$

Example 1.12 A thin rod $0 \leq x \leq L$ on the x -axis has charge density $\lambda = \lambda_0 x^2$. Find the field at the point P on the axis at distance r beyond the end $x = L$.

Solution. An element dx at x is at distance $L + r - x$ from P :

$$E = k\lambda_0 \int_0^L \frac{x^2 dx}{(L + r - x)^2}.$$

Put $u = L + r - x$ (from $L + r$ to r): $\int_r^{L+r} \frac{(L + r - u)^2}{u^2} du = \frac{L^2}{r} + 2L - 2(L + r) \ln \frac{L + r}{r}$.

Answer: $E = k\lambda_0 \left[\frac{L^2}{r} + 2L - 2(L + r) \ln \left(1 + \frac{L}{r} \right) \right]$, away from the rod

Example 1.13 A ring of radius 10 cm carries a uniform charge of 10 nC. Find the greatest field on its axis and where it occurs.

Solution. At $x = R/\sqrt{2} = 7.07$ cm: $E_{\max} = \frac{2kQ}{3\sqrt{3}R^2} = \frac{2 \times 9 \times 10^9 \times 10^{-8}}{3\sqrt{3} \times 0.01} = 3464 \text{ N C}^{-1}$.

Answer: $\approx 3.46 \times 10^3 \text{ N C}^{-1}$ at 7.07 cm from the centre

Example 1.14 A thin ring of radius $R = 50$ cm has a gap of 2 cm and carries a charge 3.13 nC spread uniformly over the remaining wire. Find the field at the centre.

Solution. Length of wire $2\pi R - w = 3.14 - 0.02 = 3.12$ m, so $\lambda = 3.13 \times 10^{-9}/3.12 \approx 1.0 \times 10^{-9} \text{ C m}^{-1}$. $E_0 = k\lambda w/R^2 = 9 \times 10^9 \times 10^{-9} \times 0.02/0.25 = 0.72 \text{ N C}^{-1}$, towards the gap.

Answer: 0.72 N C^{-1} , towards the gap

Example 1.15 A disc of radius R carries surface density σ . At what fraction of $\sigma/2\epsilon_0$ is the field on its axis at $x = R$?

Solution. $E = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{R}{\sqrt{2}R} \right) = \frac{\sigma}{2\epsilon_0} \left(1 - \frac{1}{\sqrt{2}} \right) \approx 0.29 \frac{\sigma}{2\epsilon_0}$.

Answer: $\left(1 - \frac{1}{\sqrt{2}} \right) \frac{\sigma}{2\epsilon_0}$

1.6 Electric flux and Gauss's law

Electric flux

Flux measures the field lines crossing a surface: uniform field $\phi = \vec{E} \cdot \vec{S} = ES \cos \theta$ ($\vec{S}$ along the normal); in general $\phi = \int \vec{E} \cdot d\vec{S}$. Unit $\text{N m}^2 \text{C}^{-1} = \text{V m}$. For a closed surface the normal is taken outward: entering flux is negative, leaving flux positive.

- "Flux passing through" a surface: use magnitudes. A uniform field E through a box with end faces A : $|\phi_{\text{in}}| = |\phi_{\text{out}}| = EA$.
- "Net flux emerging": add with signs. For the box, $\phi_{\text{in}} + \phi_{\text{out}} = -EA + EA = 0$.

Gauss's law

For any closed surface,

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{\text{enc}}}{\epsilon_0}.$$

Only the enclosed charge decides the flux, but $\vec{E}$ on the surface is due to *all* charges, inside and outside. The flux does not depend on the shape or size of the surface or on where the charges are inside it.

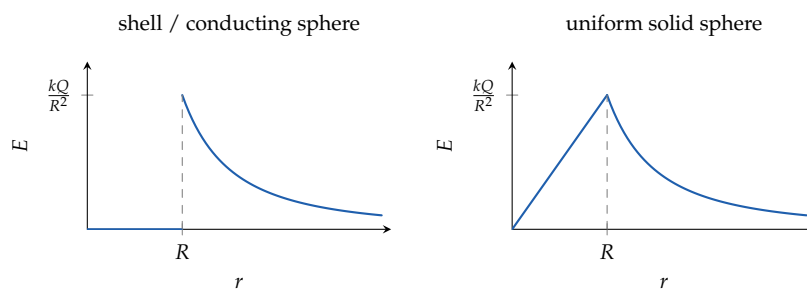
Symmetry tricks for a point charge q

- At the centre of a cube: through each face $q/6\epsilon_0$.
- At a corner of a cube: through the whole cube $q/8\epsilon_0$ (8 such cubes enclose it); through each of the 3 far faces $q/24\epsilon_0$; through the 3 faces meeting at the corner, 0.
- At the centre of a face: through the cube $q/2\epsilon_0$.
- Through a disc seen from q at a half-angle α (on its axis): $\phi = \frac{q}{2\epsilon_0} (1 - \cos \alpha)$.

Results from Gauss's law

Body	inside	surface	outside (r)
Spherical shell, or solid / hollow conducting sphere (Q, R)	0	$\frac{kQ}{R^2}$	$\frac{kQ}{r^2}$
Uniformly charged non-conducting solid sphere	$\frac{kQr}{R^3} = \frac{\rho r}{3\epsilon_0}$	$\frac{kQ}{R^2}$	$\frac{kQ}{r^2}$
Long conducting cylinder (σ)	0	$\frac{\sigma}{\epsilon_0}$	$\frac{\sigma R}{\epsilon_0 r}$
Long non-conducting cylinder (ρ)	$\frac{\rho r}{2\epsilon_0}$	$\frac{\rho R}{2\epsilon_0}$	$\frac{\rho R^2}{2\epsilon_0 r}$
Thick non-conducting slab (ρ , thickness d ; x from the mid-plane)	$\frac{\rho x}{\epsilon_0}$	$\frac{\rho d}{2\epsilon_0}$	$\frac{\rho d}{2\epsilon_0}$ (uniform)

A thick conducting slab gives the same results as a thin one ($E = 0$ inside).

**Cavities in a uniformly charged body**

A spherical cavity (centre C_2) inside a uniformly charged sphere (ρ , centre C_1) has a **uniform** field

$$\vec{E}_{\text{cavity}} = \frac{\rho}{3\epsilon_0} \vec{C_1 C_2}.$$

A cylindrical cavity inside a long uniformly charged cylinder (parallel axes): $\vec{E}_{\text{cavity}} = \frac{\rho}{2\epsilon_0} \vec{C_1 C_2}$, with $\vec{C_1 C_2}$ perpendicular to the axes. (Proof: the full body plus a body of density $-\rho$ filling the cavity.)

Example 1.16 A point charge q is at a corner of a cube. Find the flux through the cube and through one face not touching the charge.

Solution. Eight such cubes around the corner enclose q completely, so the cube gets $q/8\epsilon_0$. The three faces through the corner carry no flux ($\vec{E}$ lies in them), so each of the other three faces gets $\frac{1}{3} \cdot \frac{q}{8\epsilon_0}$.

Answer: $\frac{q}{8\epsilon_0}; \frac{q}{24\epsilon_0}$ per far face

Example 1.17 A point charge q is on the axis of a disc of radius R at distance R from it. Find the flux through the disc.

Solution. The disc subtends the half-angle $\alpha = 45^\circ$: $\phi = \frac{q}{2\epsilon_0}(1 - \cos 45^\circ) = \frac{q}{2\epsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right)$.

Answer: $\frac{q}{2\epsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right) \approx 0.146 \frac{q}{\epsilon_0}$

Example 1.18 A non-conducting sphere of radius R has charge density $\rho = \rho_0(1 + x/R)$ at distance x from its centre. Find the field at its surface and at a distance $r > R$.

Solution.

$$q_{\text{enc}} = \int_0^R \rho_0 \left(1 + \frac{x}{R}\right) 4\pi x^2 dx = 4\pi\rho_0 \left(\frac{R^3}{3} + \frac{R^3}{4}\right) = \frac{7}{3}\pi\rho_0 R^3.$$

Gauss: $E \cdot 4\pi R^2 = q_{\text{enc}}/\epsilon_0$, so

$$E(R) = \frac{7\rho_0 R}{12\epsilon_0}, \quad E(r) = \frac{7\rho_0 R^3}{12\epsilon_0 r^2} \quad (r > R).$$

Answer: $\frac{7\rho_0 R}{12\epsilon_0}$ at the surface; $\frac{7\rho_0 R^3}{12\epsilon_0 r^2}$ outside

Example 1.19 A sphere of radius R with uniform density ρ has a spherical cavity whose centre is at distance $R/2$ from the centre of the sphere. Find the field inside the cavity.

Solution. $E = \frac{\rho}{3\epsilon_0} |C_1 C_2| = \frac{\rho R}{6\epsilon_0}$, the same at every point of the cavity, along $C_1 \vec{C}_2$.

Answer: uniform, $\frac{\rho R}{6\epsilon_0}$ along the line of centres

1.7 Electric dipole

Dipole moment and field

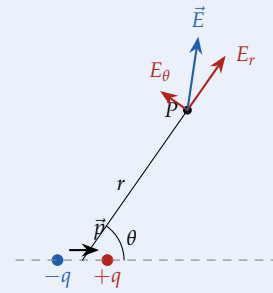
Charges $\pm q$ a small distance d apart: $\vec{p} = q\vec{d}$, directed from $-q$ to $+q$ (unit C m). At distance $r \gg d$ from the centre, at angle θ to $\vec{p}$:

$$E_r = \frac{2kp \cos \theta}{r^3}, \quad E_\theta = \frac{kp \sin \theta}{r^3}, \quad E = \frac{kp}{r^3} \sqrt{1 + 3 \cos^2 \theta},$$

and $\vec{E}$ makes angle α with the radial line, $\tan \alpha = \frac{1}{2} \tan \theta$.

- Axis ($\theta = 0$): $E = \frac{2kp}{r^3}$, along $\vec{p}$.
- Equator ($\theta = 90^\circ$): $E = \frac{kp}{r^3}$, opposite to $\vec{p}$.

The field falls as $1/r^3$ (a point charge: $1/r^2$).

**Dipole in an external field**

Uniform field: net force zero, torque $\vec{\tau} = \vec{p} \times \vec{E}$, $\tau = pE \sin \theta$. The torque turns $\vec{p}$ towards $\vec{E}$: $\theta = 0$ is stable, $\theta = 180^\circ$ unstable equilibrium. Small oscillations about $\theta = 0$ (moment of inertia I): $T = 2\pi \sqrt{I/pE}$.

Non-uniform field: there is also a net force. For $\vec{p}$ along a field that varies with r ,

$$F = p \frac{dE}{dr} \quad \left(\vec{F} = -\frac{dU}{dr} \hat{r}, \quad U = -\vec{p} \cdot \vec{E} \right),$$

towards the stronger field when $\vec{p}$ is parallel to $\vec{E}$. E.g. a dipole along the field of a point charge Q at distance r feels $F = 2kQp/r^3$.

Example 1.20 A dipole consists of ± 2 nC separated by 2 mm. Find the field at 10 cm from its centre (a) on the axis, (b) on the equator.

Solution. $p = 2 \times 10^{-9} \times 2 \times 10^{-3} = 4 \times 10^{-12}$ C m. $\frac{kp}{r^3} = \frac{9 \times 10^9 \times 4 \times 10^{-12}}{10^{-3}} = 36$ NC $^{-1}$. So $E_{\text{axis}} = 72$ NC $^{-1}$ along $\vec{p}$ and $E_{\text{eq}} = 36$ NC $^{-1}$ opposite to $\vec{p}$.

Answer: (a) 72 NC $^{-1}$ (b) 36 NC $^{-1}$

Example 1.21 At what angle θ from the axis of a short dipole is the field perpendicular to the axis?

Solution. The component of $\vec{E}$ along $\vec{p}$ is $E_r \cos \theta - E_\theta \sin \theta \propto 2 \cos^2 \theta - \sin^2 \theta = 0$, so $\tan^2 \theta = 2$.

Answer: $\theta = \tan^{-1} \sqrt{2} \approx 54.7^\circ$

Example 1.22 A dipole of moment 4×10^{-9} C m is at 30° to a uniform field of 5×10^4 NC $^{-1}$. Find the torque on it.

Solution. $\tau = pE \sin 30^\circ = 4 \times 10^{-9} \times 5 \times 10^4 \times 0.5 = 10^{-4}$ N m, turning $\vec{p}$ towards $\vec{E}$.

Answer: 1×10^{-4} N m