



Continuity and Differentiability — CBSE Class 12 Mathematics Chapter Test

FREE CHAPTER TEST · CLASS XII · MATHEMATICS (041) · PATTERN OF THE CBSE SAMPLE QUESTION PAPER 2026-27

TIME ALLOWED	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
1½ hours	40	19 (A 10 · B 3 · C 2 · D 2 · E 2)	

GENERAL INSTRUCTIONS

Read the following instructions very carefully and strictly follow them:

1. This question paper contains 19 questions. All questions are compulsory.
2. This question paper is divided into five sections: A, B, C, D and E.
3. In Section A, Questions 1 to 8 are multiple choice questions (MCQs) and Questions 9 and 10 are assertion-reason based questions of 1 mark each.
4. In Section B, Q11–13 are very short answer (VSA) type questions, carrying 2 marks each.
5. In Section C, Q14–15 are short answer (SA) type questions, carrying 3 marks each.
6. In Section D, Q16–17 are long answer (LA) type questions, carrying 5 marks each.
7. In Section E, Q18–19 are case study based questions, carrying 4 marks each, with sub-parts of the values of 1, 1 and 2 marks.
8. There is no overall choice. However, an internal choice has been provided in 1 question of Section B, 1 question of Section C, 1 question of Section D and in the 2-mark sub-part of each question of Section E.
9. Use of calculators is not allowed. $\log x$ denotes the natural logarithm of x .
10. This chapter test follows the section structure, question types and marks per question of the CBSE Class XII Mathematics (041) Sample Question Paper 2026-27, released on 21 September 2026 at cbseacademic.nic.in, at half the length of the 80-mark board paper: 40 marks in 1½ hours.
11. The answer key and the marking scheme are at the end of this booklet. Attempt the paper in 1½ hours before you look at them.

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Chapter: Continuity and Differentiability · **Topics:** Continuity · Differentiability · Chain rule · Derivatives of inverse trigonometric functions · Implicit and parametric functions · Exponential and logarithmic functions · Logarithmic differentiation · Second order derivatives

SECTION A

This section comprises 10 questions of 1 mark each: 8 multiple choice questions (MCQs) and 2 assertion-reason based questions.

1. If the function $f(x) = \begin{cases} \frac{e^{5x} - e^{2x}}{\log(1 + 6x)}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then k equals 1
 - (A) $\frac{1}{2}$ (B) 2
 - (C) $\frac{7}{6}$ (D) 3

2. The number of points at which $f(x) = \min \{|x|, |x - 2|, 1\}$, $x \in \mathbb{R}$, is not differentiable is 1
 - (A) 3 (B) 4
 - (C) 5 (D) 6

3. For $-\frac{1}{2} < x < \frac{1}{2}$, the derivative of $\cos^{-1}(4x^3 - 3x)$ with respect to x is 1
 - (A) $-\frac{3}{\sqrt{1-x^2}}$ (B) $\frac{1}{\sqrt{1-x^2}}$
 - (C) $-\frac{1}{\sqrt{1-x^2}}$ (D) $\frac{3}{\sqrt{1-x^2}}$

4. If $x = e^t \cos t$ and $y = e^t \sin t$, then $\frac{dy}{dx}$ at $t = \frac{\pi}{12}$ is 1
 - (A) $\frac{1}{\sqrt{3}}$ (B) $\sqrt{3}$
 - (C) $2 + \sqrt{3}$ (D) 1

5. If $y = x^x$, $x > 0$, then the value of $\frac{d^2y}{dx^2}$ at $x = 1$ is 1
 - (A) 0 (B) 1
 - (C) 2 (D) 3

6. Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Then, at $x = 0$, f is 1
 - (A) not continuous (B) continuous but not differentiable
 - (C) differentiable, with $f'(0) = 1$ (D) differentiable, with $f'(0) = 0$

7. If $y = (1 + x)(1 + x^2)(1 + x^4)(1 + x^8)$, then $\frac{dy}{dx}$ at $x = 1$ is 1
 - (A) 120 (B) 136
 - (C) 64 (D) 128

8. If $f(x) = \begin{cases} ax^2 + bx, & x \leq 1 \\ 2 \log x + 3, & x > 1 \end{cases}$ is differentiable at $x = 1$, then 1
- (A) $a = 1, b = 2$ (B) $a = -1, b = 4$
 (C) $a = -1, b = 2$ (D) $a = 2, b = 1$
9. **Assertion (A):** The function $f(x) = |x| + |x - 1|$ is continuous at every real number but is not differentiable at exactly two points. 1
Reason (R): The sum of two functions that are continuous at a point is continuous at that point.
- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false.
 (D) Assertion (A) is false, but Reason (R) is true.
10. **Assertion (A):** The function $f(x) = |\sin x|$ is not differentiable at $x = \pi$. 1
Reason (R): A function that is continuous at a point is differentiable at that point.
- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
 (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
 (C) Assertion (A) is true, but Reason (R) is false.
 (D) Assertion (A) is false, but Reason (R) is true.

SECTION B

This section comprises 3 very short answer (VSA) type questions of 2 marks each.

11. Find the value of k for which $f(x) = \begin{cases} \frac{\sin 3x - 3 \sin x}{x^3}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$. 2
12. (a) Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left(\frac{x + \sqrt{1-x^2}}{\sqrt{2}}\right)$, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$. 2
- OR**
- (b) If $x = e^t + e^{-t}$ and $y = e^t - e^{-t}$, find $\frac{d^2y}{dx^2}$ in terms of y .
13. Find all the points at which $f(x) = |x^2 - 9| + |x - 3|$ is not differentiable. 2

SECTION C

This section comprises 2 short answer (SA) type questions of 3 marks each.

14. (a) If $y = x \sin(\log x)$, $x > 0$, prove that $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + 2y = 0$. 3

OR

- (b) If $y = (1 + x^2)^x + x^{1/x}$, $x > 0$, find $\frac{dy}{dx}$ at $x = 1$.

15. Find the values of a and b for which the following function is continuous at $x = 0$: $f(x) =$ 3

$$\begin{cases} \frac{1 - \cos 4x}{8x^2}, & x < 0 \\ a, & x = 0 \\ \frac{b(\sqrt{x+4} - 2)}{x}, & x > 0 \end{cases}$$

SECTION D

This section comprises 2 long answer (LA) type questions of 5 marks each.

16. (a) Find the values of a , b , c and d for which the following function is differentiable at every real 5

$$\text{number } x: f(x) = \begin{cases} a \sin x + b \cos x, & x \leq 0 \\ x^2 + cx + d, & 0 < x < 2 \\ 4 \log(x - 1) + 6x - 6, & x \geq 2 \end{cases}$$

OR

- (b) If $x = t^3 - 3t$ and $y = 3t^2$ ($t \neq \pm 1$), find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of t . Hence find the value of $\frac{d^2y}{dx^2}$ at $t = 2$, and show that $\frac{d^2y}{dx^2} < 0$ whenever $|t| > 1$.

17. If $y = \log\left(\frac{x^2 + x + 1}{x^2 - x + 1}\right) + \frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)\right]$, prove that $\frac{dy}{dx} = \frac{4}{x^4 + x^2 + 1}$. 5

SECTION E

This section comprises 2 case study based questions of 4 marks each, with sub-parts of 1, 1 and 2 marks.

18. A delivery drone takes off vertically. Its height h (in metres) t seconds after take-off is modelled by $h(t)$

$$= \begin{cases} 3t^2, & 0 \leq t \leq 2 \\ at + b, & 2 < t \leq 6 \\ 72 - 3(t - 8)^2, & 6 < t \leq 8 \end{cases} \quad \text{For a smooth flight there must be no sudden jump in the height or in}$$

the vertical velocity, i.e. h must be continuous and differentiable.

- (i) Write the condition on a and b for h to be continuous at $t = 2$. [1]
(ii) Find a and b if h is also differentiable at $t = 2$. [1]
(iii) With these values, show that h is continuous and differentiable at $t = 6$, and find the vertical velocity $h'(6)$. [2]

OR

- (iii) Show that $h'(t)$ is not differentiable at $t = 2$, that is, $h''(2)$ does not exist. [2]

19. A cycling track is built in the shape of an ellipse. With the centre of the track as origin, a cyclist's position on the upper half of the track is $x = 40 \cos t$, $y = 30 \sin t$ (in metres), $0 < t < \pi$.

- (i) Find $\frac{dy}{dx}$ in terms of t . [1]
(ii) Find the point of the upper half of the track at which $\frac{dy}{dx} = -\frac{3}{4}$. [1]
(iii) Find $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$. [2]

OR

- (iii) Show that $y^3 \frac{d^2y}{dx^2} = -\frac{2025}{4}$ at every point of the upper half of the track. [2]

END OF THE QUESTION PAPER

ANSWER KEY — SECTION A

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(A)	(C)	(D)	(B)	(C)	(D)	(A)	(B)	(B)	(C)

MARKING SCHEME AND WORKED SOLUTIONS

Marks for each step are shown in brackets, as in the CBSE marking scheme. Any correct alternative method earns full marks.

Q1. Answer (A). $k = \lim_{x \rightarrow 0} \frac{e^{5x} - e^{2x}}{\log(1 + 6x)} = \lim \left[\frac{e^{5x} - 1}{x} - \frac{e^{2x} - 1}{x} \right] \div \frac{\log(1 + 6x)}{x} = (5 - 2)/6 = 1/2.$

Q2. Answer (C). For $-1 \leq x \leq 1$, $f(x) = |x|$; for $1 \leq x \leq 3$, $f(x) = |x - 2|$; for $x \leq -1$ or $x \geq 3$, $f(x) = 1$. The graph has corners at $x = -1, 0, 1, 2$ and 3 (at $x = 1$ the slope changes from $+1$ to -1). So f is not differentiable at 5 points.

Q3. Answer (D). Put $x = \cos \theta$. For $-1/2 < x < 1/2$, $\theta \in (\pi/3, 2\pi/3)$, so $3\theta \in (\pi, 2\pi)$ and $\cos^{-1}(\cos 3\theta) = 2\pi - 3\theta$. Hence $y = 2\pi - 3 \cos^{-1}x$ and $dy/dx = 3/\sqrt{1 - x^2}$. (The answer $-3/\sqrt{1 - x^2}$ holds only for $1/2 < x < 1$.)

Q4. Answer (B). $dx/dt = e^t(\cos t - \sin t)$, $dy/dt = e^t(\sin t + \cos t)$, so $dy/dx = (1 + \tan t)/(1 - \tan t) = \tan(\pi/4 + t)$. At $t = \pi/12$ this is $\tan(\pi/3) = \sqrt{3}$.

Q5. Answer (C). $\log y = x \log x$ gives $y' = x^x(1 + \log x)$. Then $y'' = x^x(1 + \log x)^2 + x^x \cdot (1/x)$. At $x = 1$: $y'' = 1 \cdot 1 + 1 \cdot 1 = 2$.

Q6. Answer (D). $|x^2 \sin(1/x)| \leq x^2 \rightarrow 0$, so f is continuous at 0. $[f(h) - f(0)]/h = h \sin(1/h)$, and $|h \sin(1/h)| \leq |h| \rightarrow 0$, so $f'(0)$ exists and equals 0.

Q7. Answer (A). Multiplying out, $y = 1 + x + x^2 + \dots + x^{15}$ (since $(1 - x)y = 1 - x^{16}$). So dy/dx at $x = 1$ is $1 + 2 + \dots + 15 = 120$.

Q8. Answer (B). Continuity at 1: $a + b = 2 \log 1 + 3 = 3$. Left derivative $2a + b$, right derivative $2/x$ at $x = 1$, i.e. 2. So $2a + b = 2$, giving $a = -1$, $b = 4$. ($a = 1$, $b = 2$ makes f continuous but not differentiable at 1.)

Q9. Answer (B). f is continuous everywhere, and its slope is -2 for $x < 0$, 0 for $0 < x < 1$ and 2 for $x > 1$, so it has corners at $x = 0$ and $x = 1$ only: A is true. R is true, but it explains only the continuity, not why f fails to be differentiable at exactly two points. Answer (B).

Q10. Answer (C). Near $x = \pi$, $|\sin x| = \sin x$ for $x < \pi$ and $-\sin x$ for $x > \pi$, so the left derivative is $\cos \pi = -1$ and the right derivative is $+1$: A is true. R is false: $|x|$ is continuous at 0 but not differentiable there. Answer (C).

Q11. (2 marks) Answer: $k = -4$.

$\sin 3x = 3 \sin x - 4 \sin^3 x$, so the numerator is $-4 \sin^3 x$. [1]

$\lim_{x \rightarrow 0} f(x) = -4 \lim (\sin x / x)^3 = -4$. For continuity, $k = f(0) = -4$. [1]

Q12. (2 marks) Answer: (a) $1/\sqrt{1 - x^2}$; (b) $-4/y^3$.

(a) Put $x = \sin \theta$, $\theta \in (-\pi/4, \pi/4)$. Then $(x + \sqrt{1 - x^2})/\sqrt{2} = \sin \theta \cos(\pi/4) + \cos \theta \sin(\pi/4) = \sin(\theta + \pi/4)$, with $\theta + \pi/4 \in (0, \pi/2)$. [1]

So $y = \theta + \pi/4 = \sin^{-1}x + \pi/4$ and $dy/dx = 1/\sqrt{1 - x^2}$. [1]

OR (b) $dx/dt = e^t - e^{-t} = y$ and $dy/dt = e^t + e^{-t} = x$, so $dy/dx = x/y$. [1]

Also $x^2 - y^2 = 4$. Then $d^2y/dx^2 = (y - x \cdot dy/dx)/y^2 = (y^2 - x^2)/y^3 = -4/y^3$. [1]

Q13. (2 marks) Answer: $x = -3$ and $x = 3$.

Both terms are differentiable except where $x^2 - 9 = 0$ or $x - 3 = 0$, i.e. possibly at $x = -3$ and $x = 3$. [$\frac{1}{2}$]

At $x = 3$: left derivative $-6 - 1 = -7$, right derivative $6 + 1 = 7$, so f is not differentiable. [$\frac{1}{2}$]

At $x = -3$: $|x - 3|$ is differentiable there (slope -1) but $|x^2 - 9|$ has left slope -6 and right slope $+6$, so f has left derivative -7 and right derivative 5 : not differentiable. [$\frac{1}{2}$]

f is not differentiable at $x = -3$ and $x = 3$ only. [$\frac{1}{2}$]

Q14. (3 marks) Answer: (a) proved; (b) $3 + 2 \log 2$.

(a) $dy/dx = \sin(\log x) + x \cos(\log x) \cdot (1/x) = \sin(\log x) + \cos(\log x)$. [1]

$d^2y/dx^2 = [\cos(\log x) - \sin(\log x)]/x$. [1]

$x^2y'' - xy' + 2y = x \cos(\log x) - x \sin(\log x) - x \sin(\log x) - x \cos(\log x) + 2x \sin(\log x) = 0$. [1]

OR (b) Let $u = (1 + x^2)^x$: $\log u = x \log(1 + x^2)$, so $u' = (1 + x^2)^x [\log(1 + x^2) + 2x^2/(1 + x^2)]$ and $u'(1) = 2(\log 2 + 1)$. [1½]

Let $v = x^{1/x}$: $\log v = (\log x)/x$, so $v' = x^{1/x} \cdot (1 - \log x)/x^2$ and $v'(1) = 1$. [1]

dy/dx at $x = 1 = 2 + 2 \log 2 + 1 = 3 + 2 \log 2$. [½]

Q15. (3 marks) Answer: $a = 1$, $b = 4$.

Left-hand limit: $(1 - \cos 4x)/(8x^2) = 2 \sin^2 2x/(8x^2) = (\sin 2x / 2x)^2 \rightarrow 1$. [1]

Right-hand limit: $b(\sqrt{x+4} - 2)/x = b/(\sqrt{x+4} + 2) \rightarrow b/4$. [1]

Continuity: $1 = a = b/4$, so $a = 1$ and $b = 4$. [1]

Q16. (5 marks) Answer: (a) $a = 6$, $b = -10$, $c = 6$, $d = -10$; (b) $-10/81$.

(a) Each piece is differentiable on its own open interval, so only $x = 0$ and $x = 2$ need checking. [½]

At $x = 0$: continuity gives $b = d$; left derivative $a \cos 0 - b \sin 0 = a$, right derivative $2 \cdot 0 + c = c$, so $a = c$. [1½]

At $x = 2$: continuity gives $4 + 2c + d = 4 \log 1 + 12 - 6 = 6$, i.e. $2c + d = 2$; left derivative $4 + c$, right derivative $4/(2 - 1) + 6 = 10$, so $c = 6$. [1½]

Then $d = 2 - 12 = -10$, $a = c = 6$ and $b = d = -10$. [1]

$a = 6$, $b = -10$, $c = 6$, $d = -10$. [½]

OR (b) $dx/dt = 3t^2 - 3$, $dy/dt = 6t$, so $dy/dx = 6t/(3t^2 - 3) = 2t/(t^2 - 1)$. [1½]

$d/dt(dy/dx) = [2(t^2 - 1) - 2t \cdot 2t]/(t^2 - 1)^2 = -2(t^2 + 1)/(t^2 - 1)^2$. [1½]

$d^2y/dx^2 = [-2(t^2 + 1)/(t^2 - 1)^2] \div [3(t^2 - 1)] = -2(t^2 + 1)/[3(t^2 - 1)^3]$. [1]

At $t = 2$: $-2 \cdot 5/(3 \cdot 27) = -10/81$. For $|t| > 1$, $(t^2 - 1)^3 > 0$ and $t^2 + 1 > 0$, so $d^2y/dx^2 < 0$. [1]

Q17. (5 marks) Answer: proved.

$d/dx \log[(x^2 + x + 1)/(x^2 - x + 1)] = (2x + 1)/(x^2 + x + 1) - (2x - 1)/(x^2 - x + 1)$. [1]

$d/dx (2/\sqrt{3}) \tan^{-1}((2x + 1)/\sqrt{3}) = (2/\sqrt{3}) \cdot (2/\sqrt{3})/[1 + (2x + 1)^2/3] = 4/[3 + (2x + 1)^2] = 1/(x^2 + x + 1)$; similarly the last term gives $1/(x^2 - x + 1)$. [2]

So $dy/dx = (2x + 2)/(x^2 + x + 1) + (2 - 2x)/(x^2 - x + 1)$. [½]

Numerator over $(x^2 + x + 1)(x^2 - x + 1) = x^4 + x^2 + 1$: $(2x + 2)(x^2 - x + 1) + (2 - 2x)(x^2 + x + 1) = (2x^3 + 2) + (2 - 2x^3) = 4$. [1]

Hence $dy/dx = 4/(x^4 + x^2 + 1)$. [½]

Q18. (4 marks) Answer: (i) $2a + b = 12$; (ii) $a = 12$, $b = -12$; (iii)(a) $h'(6) = 12$ m/s.

(i) $h(2) = 12$ and the right-hand limit is $2a + b$, so $2a + b = 12$. [1]

(ii) Left derivative at 2: $6t = 12$; right derivative: a . So $a = 12$ and $b = 12 - 24 = -12$. [1]

(iii)(a) At $t = 6$: $12 \cdot 6 - 12 = 60$ and $72 - 3(6 - 8)^2 = 60$, so h is continuous. [1]

Left derivative 12; right derivative $-6(t - 8) = 12$ at $t = 6$. So h is differentiable at 6 and $h'(6) = 12$ m/s. [1]

OR (iii)(b) $h'(t) = 6t$ for $0 < t < 2$, $h'(2) = 12$, and $h'(t) = 12$ for $2 < t < 6$. [1]

Left derivative of h' at 2 is 6 and the right derivative is 0. They differ, so $h''(2)$ does not exist. [1]

Q19. (4 marks) Answer: (i) $-(3/4) \cot t$; (ii) $(20\sqrt{2}, 15\sqrt{2})$; (iii)(a) $-3\sqrt{2}/80$.

(i) $dx/dt = -40 \sin t$, $dy/dt = 30 \cos t$, so $dy/dx = -(3/4) \cot t$. [1]

(ii) $-(3/4) \cot t = -3/4$ gives $\cot t = 1$, $t = \pi/4$: the point $(20\sqrt{2}, 15\sqrt{2})$. [1]

(iii)(a) $d/dt(-(3/4) \cot t) = (3/4) \operatorname{cosec}^2 t$, so $d^2y/dx^2 = (3/4) \operatorname{cosec}^2 t \div (-40 \sin t) = -(3/160) \operatorname{cosec}^3 t$. [1]

At $t = \pi/4$, $\operatorname{cosec}^3 t = 2\sqrt{2}$, so $d^2y/dx^2 = -3\sqrt{2}/80$. [1]

OR (iii)(b) As in (a), $d^2y/dx^2 = -3/(160 \sin^3 t)$. [1]

$y^3 = 27000 \sin^3 t$, so $y^3 \cdot d^2y/dx^2 = -27000 \cdot 3/160 = -2025/4$. [1]