



Algebra — JEE (Advanced) Pattern Sample Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (ADVANCED) PAPER 1

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	60	17 (4 + 3 + 6 + 4)	

GENERAL INSTRUCTIONS

1. This question paper has FOUR sections. All questions are compulsory.
2. Read the marking scheme of each section carefully: wrong answers in Sections 1, 2 and 4 carry negative marks.
3. Use of a calculator, log tables or any other aid is not allowed.
4. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

SECTION 1 (Q1–4) — +3 / 0 / –1

1. This section contains FOUR (04) questions.
2. Each question has FOUR options. ONLY ONE of these four options is the correct answer.
3. Full Marks: +3 if ONLY the correct option is chosen; Zero Marks: 0 if none of the options is chosen; Negative Marks: –1 in all other cases.

SECTION 2 (Q5–7) — +4 / partial / –2

1. This section contains THREE (03) questions.
2. Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
3. Full Marks: +4 ONLY if (all) the correct option(s) is(are) chosen; Partial Marks: +3 if all the four options are correct but ONLY three options are chosen; +2 if three or more options are correct but ONLY two options are chosen, both of which are correct; +1 if two or more options are correct but ONLY one option is chosen and it is a correct option; Zero Marks: 0 if unanswered; Negative Marks: –2 in all other cases.

SECTION 3 (Q8–13) — +4 / 0

1. This section contains SIX (06) questions.
2. The answer to each question is a NON-NEGATIVE INTEGER.
3. Full Marks: +4 ONLY if the correct integer is entered; Zero Marks: 0 in all other cases.

SECTION 4 (Q14–17) — +3 / 0 / –1

1. This section contains FOUR (04) Matching List Sets.
2. Each set has ONE Multiple Choice Question. Each set has TWO lists: List-I and List-II. List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
3. FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
4. Full Marks: +3 ONLY if the option corresponding to the correct combination is chosen; Zero Marks: 0 if none of the options is chosen; Negative Marks: –1 in all other cases.

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Chapters covered: Logarithm · Quadratic Equation · Sequence and Progression · Permutation and Combination · Binomial Theorem · Determinants and Matrices · Probability · Complex Number

SECTION 1 (Q1–4) · Single correct · +3 / 0 / –1

Each question has FOUR options. ONLY ONE of these four options is the correct answer.

1. A complex number z moves so that $|z - 2i| + |z + 2i| = 6$. The greatest value of $|z - 3|$ is
- (A) 6
(B) $3 + \sqrt{5}$
(C) $\sqrt{34}$
(D) $2 + \sqrt{5}$
2. The sum of the infinite series $\sum (k^2 + k - 1)/(k + 2)!$, taken over $k = 1, 2, 3, \dots$, is
- (A) $e - 2$
(B) 1
(C) $e/2 - 1$
(D) $1/2$
3. If $(1 + x)^{20} = C_0 + C_1x + C_2x^2 + \dots + C_{20}x^{20}$, then $C_0 - C_2 + C_4 - C_6 + \dots + C_{20}$ equals
- (A) -1024
(B) 1024
(C) 0
(D) 512
4. Box I has 3 red and 2 black balls; Box II has 2 red and 3 black balls. A fair die is rolled: if it shows 1 or 2, Box I is chosen, otherwise Box II. Two balls are drawn together from the chosen box and both are red. The probability that Box I was chosen is
- (A) $1/2$
(B) $3/10$
(C) $3/5$
(D) $2/5$

SECTION 2 (Q5–7) · One or more correct · +4 / partial / –2

Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).

5. Let M be the 3×3 matrix whose diagonal entries are all 1 and whose other entries are all 2, and let I be the 3×3 identity matrix. Which of the following is/are correct?
- (A) $M^2 = 4M + 5I$
(B) $\det M = -5$
(C) $M^{-1} = (M - 4I)/5$
(D) $\det(\text{adj}(\text{adj } M)) = 625$
6. Consider the equation $x^2 - 2(a - 1)x + 2a + 1 = 0$, where a is a real number. Which of the following is/are correct?
- (A) Both roots are positive if and only if $a \geq 4$.
(B) The roots are real and of opposite signs if and only if $a < -1/2$.
(C) Both roots are negative if and only if $a \leq 0$.
(D) When $a = -1$, exactly one root lies in the interval $(1, 2)$.

7. Five distinct balls are to be placed in boxes, any number of balls in a box. Which of the following is/are correct?
- (A) The number of ways of placing them in three distinct boxes with no box empty is 120.
- (B) The number of ways of placing them in three identical boxes with no box empty is 25.
- (C) The number of ways of placing them in three distinct boxes with exactly one box empty is 90.
- (D) If each ball is placed in one of three distinct boxes at random, the probability that a particular box gets exactly two balls is $\frac{80}{243}$.

SECTION 3 (Q8–13) · Non-negative integer · +4 / 0

The answer to each question is a NON-NEGATIVE INTEGER.

8. The set of all real values of a for which the equation $(\log_2 x)^2 - a \cdot \log_2 x + 4 = 0$ has two distinct solutions in the interval $(1, 64)$ is an open interval (p, q) . Find the value of $3(p + q)$.

Answer: _____

9. Let x , y and z be positive real numbers with $x + 2y + 3z = 12$. Find the greatest value of $x y^2 z^3$.

Answer: _____

10. Find the coefficient of x^7 in the expansion of $(1 + x + x^2)^7 (1 - x)^6$.

Answer: _____

11. Find the number of ordered pairs (z, w) of complex numbers that satisfy both $z \cdot w^5 = 1$ and $z^3 \cdot w = i$.

Answer: _____

12. Find the number of 2×2 matrices, each of whose four entries is -1 , 0 or 1 , whose determinant is zero.

Answer: _____

13. Three different numbers are chosen at random from $\{1, 2, 3, \dots, 20\}$. If p is the probability that they form an arithmetic progression (in some order), find $38p$.

Answer: _____

SECTION 4 (Q14–17) · Matching lists · +3 / 0 / -1

Each set has ONE Multiple Choice Question. Each set has TWO lists: List-I and List-II. List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).

14. Consider the equation $x^2 - 2ax + a^2 + a - 3 = 0$, where a is an integer with $-5 \leq a \leq 5$. Match each condition in List-I with the number of such integers a in List-II.

List-I	List-II
(P) The roots are real and both are less than 3	(1) 1
(Q) The roots are real and both are greater than 0	(2) 2
(R) -1 lies strictly between the roots	(3) 4
(S) The roots are real and both lie in the interval $(-2, 2)$	(4) 7
	(5) 5

- (A) $P \rightarrow 5, Q \rightarrow 2, R \rightarrow 3, S \rightarrow 1$
- (B) $P \rightarrow 4, Q \rightarrow 1, R \rightarrow 3, S \rightarrow 2$
- (C) $P \rightarrow 4, Q \rightarrow 2, R \rightarrow 3, S \rightarrow 1$
- (D) $P \rightarrow 4, Q \rightarrow 2, R \rightarrow 1, S \rightarrow 3$

15. Match each quantity in List-I with its value in List-II.

List-I	List-II
(P) The number of rational terms in the expansion of $(\sqrt{2} + \sqrt[3]{5})^{60}$	(1) 21
(Q) The remainder when 2^{2025} is divided by 33	(2) 2047/11
(R) The number of distinct terms, after like powers are collected, in the expansion of $(x + 1/x + x^2 + 1/x^2)^5$	(3) 11
(S) The value of $\sum {}^{10}C_r/(r + 1)$, taken over $r = 0, 1, \dots, 10$	(4) 32
	(5) 17

- (A) $P \rightarrow 3, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 2$
 (B) $P \rightarrow 5, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 2$
 (C) $P \rightarrow 3, Q \rightarrow 4, R \rightarrow 5, S \rightarrow 2$
 (D) $P \rightarrow 3, Q \rightarrow 1, R \rightarrow 4, S \rightarrow 2$

16. Match each counting problem in List-I with its answer in List-II.

List-I	List-II
(P) The number of ways of putting 10 identical balls into 4 distinct boxes so that every box gets at least 1 and at most 5 balls	(1) 44
(Q) The number of ways of putting 5 letters into their 5 addressed envelopes, one in each, so that no letter is in its own envelope	(2) 126
(R) The number of four-digit numbers whose digits are strictly increasing from left to right	(3) 1800
(S) The number of arrangements of all the letters of ALGEBRA in which the two A's are not next to each other	(4) 68
	(5) 84

- (A) $P \rightarrow 5, Q \rightarrow 1, R \rightarrow 2, S \rightarrow 3$
 (B) $P \rightarrow 4, Q \rightarrow 1, R \rightarrow 3, S \rightarrow 2$
 (C) $P \rightarrow 1, Q \rightarrow 4, R \rightarrow 2, S \rightarrow 3$
 (D) $P \rightarrow 4, Q \rightarrow 1, R \rightarrow 2, S \rightarrow 3$

17. Match each probability in List-I with its value in List-II.

List-I	List-II
(P) A fair coin is tossed until two consecutive heads appear: the probability that exactly 5 tosses are needed	(1) 6/11
(Q) Three fair dice are thrown: the probability that the greatest number shown is 4	(2) 3/32
(R) Balls are drawn one by one without replacement from a bag of 4 white and 6 black balls: the probability that the third white ball appears on the 5th draw	(3) 37/216
(S) A and B throw a fair die alternately, A first; whoever first throws a 6 wins: the probability that A wins	(4) 1/7
	(5) 5/11

- (A) $P \rightarrow 2, Q \rightarrow 3, R \rightarrow 4, S \rightarrow 5$
 (B) $P \rightarrow 2, Q \rightarrow 3, R \rightarrow 4, S \rightarrow 1$
 (C) $P \rightarrow 4, Q \rightarrow 3, R \rightarrow 2, S \rightarrow 1$
 (D) $P \rightarrow 2, Q \rightarrow 1, R \rightarrow 4, S \rightarrow 3$

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
B	D	A	C	A, C, D	A, B	B, C, D	32	64

Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17
15	14	33	3	C	A	D	B

WORKED SOLUTIONS

Q1. Answer B. The locus is an ellipse with foci $\pm 2i$ and major axis 6 along the imaginary axis: $x^2/5 + y^2/9 = 1$. With $x = \sqrt{5} \cos t$, $y = 3 \sin t$, $|z - 3|^2 = (\sqrt{5} \cos t - 3)^2 + 9 \sin^2 t = 18 - 6\sqrt{5} \cos t - 4 \cos^2 t$. On $[-1, 1]$ this is greatest at $\cos t = -1$ (the vertex of the parabola, $\cos t = -3\sqrt{5}/4$, lies outside), giving $14 + 6\sqrt{5} = (3 + \sqrt{5})^2$. So the greatest value is $3 + \sqrt{5}$, at $z = -\sqrt{5}$.

Q2. Answer D. Write the numerator in falling factorials of $k + 2$: $k^2 + k - 1 = (k + 2)(k + 1) - 2(k + 2) + 1$. So the k -th term is $1/k! - 2/(k + 1)! + 1/(k + 2)!$. Summing from $k = 1$: $\sum 1/k! = e - 1$, $\sum 1/(k + 1)! = e - 2$, $\sum 1/(k + 2)! = e - 5/2$. Total $= (e - 1) - 2(e - 2) + (e - 5/2) = 1/2$. The e 's cancel — a choice with e in it comes from dropping one of the shifted starting terms.

Q3. Answer A. Put $x = i$: $(1 + i)^{20} = \sum C_r i^r$, whose real part is $C_0 - C_2 + C_4 - \dots + C_{20}$ (the signs follow $i^0, i^2, i^4, \dots = 1, -1, 1, \dots$). Now $(1 + i)^2 = 2i$, so $(1 + i)^{20} = (2i)^{10} = 2^{10} \cdot i^{10} = -1024$, which is real. The answer is -1024 ; $+1024$ forgets that $i^{10} = -1$.

Q4. Answer C. $P(\text{Box I}) = 1/3$, $P(\text{Box II}) = 2/3$. $P(\text{two red} | \text{I}) = {}^3C_2/{}^5C_2 = 3/10$; $P(\text{two red} | \text{II}) = {}^2C_2/{}^5C_2 = 1/10$. By Bayes' theorem, $P(\text{I} | \text{two red}) = (1/3)(3/10) / [(1/3)(3/10) + (2/3)(1/10)] = (3/30)/(5/30) = 3/5$.

Q5. Answer A, C, D. Write $M = 2J - I$, where J is the matrix of all 1s; $J^2 = 3J$. Then $M^2 = 4J^2 - 4J + I = 8J + I$, and $4M + 5I = 8J - 4I + 5I = 8J + I$, so (A) is true. $\det M = 1(1 - 4) - 2(2 - 4) + 2(4 - 2) = -3 + 4 + 4 = 5$, so (B) is false. From $M^2 - 4M = 5I$, $M(M - 4I) = 5I$, so $M^{-1} = (M - 4I)/5$: (C) true. For an $n \times n$ matrix, $\det(\text{adj}(\text{adj } M)) = (\det M)^{((n-1)^2)} = 5^4 = 625$: (D) true.

Q6. Answer A, B. Discriminant/4 $= (a - 1)^2 - (2a + 1) = a(a - 4)$, so the roots are real iff $a \leq 0$ or $a \geq 4$. Sum $= 2(a - 1)$, product $= 2a + 1$. (A) Both positive: real, sum > 0 ($a > 1$) and product > 0 ($a > -1/2$), i.e. $a \geq 4$ — true ($a = 4$ gives the double root 3). (B) Opposite signs iff product < 0 , i.e. $a < -1/2$, and the roots are then automatically real — true. (C) Both negative needs sum < 0 ($a < 1$) and product > 0 ($a > -1/2$) as well as $a \leq 0$, i.e. $-1/2 < a \leq 0$, not all $a \leq 0$ — false ($a = -1$ gives roots of opposite signs). (D) For $a = -1$: $x^2 + 4x - 1 = 0$, roots $-2 \pm \sqrt{5} \approx 0.24$ and -4.24 , neither in $(1, 2)$; indeed $f(1) = 4$ and $f(2) = 11$ are both positive — false.

Q7. Answer B, C, D. (A) By inclusion–exclusion, $3^5 - 3 \cdot 2^5 + 3 \cdot 1^5 = 243 - 96 + 3 = 150$, not 120 — false. (B) Identical boxes: divide by $3! = 6$ (the boxes then hold distinguishable groups): $150/6 = 25$ — true. (C) Choose the empty box (3 ways), then fill the other two with neither empty: $2^5 - 2 = 30$; $3 \times 30 = 90$ — true. (D) ${}^5C_2 \cdot 2^3 / 3^5 = 10 \cdot 8/243 = 80/243$ — true.

Q8. Answer 32. Put $t = \log_2 x$; it maps $(1, 64)$ one-to-one onto $(0, 6)$, so we need $g(t) = t^2 - at + 4$ to have two distinct roots in $(0, 6)$. Conditions: discriminant $a^2 - 16 > 0$ ($a > 4$ or $a < -4$); vertex $a/2$ in $(0, 6)$, so $0 < a < 12$; $g(0) = 4 > 0$ (always); $g(6) = 40 - 6a > 0$, so $a < 20/3$. Together $4 < a < 20/3$, so $p = 4$, $q = 20/3$ and $3(p + q) = 12 + 20 = 32$.

Q9. Answer 64. Split the sum into six parts with the powers as weights: $x + 2y + 3z = x + y + y + z + z + z$. By AM $\geq$ GM on these six numbers, $12/6 \geq (x \cdot y \cdot y \cdot z \cdot z \cdot z)^{1/6}$, so $x y^2 z^3 \leq 2^6 = 64$, with equality when $x = y = z = 2$ (and $2 + 4 + 6 = 12$). Applying AM–GM to $x, 2y, 3z$ instead gives a bound that is never reached.

Q10. Answer 15. $(1 + x + x^2)(1 - x) = 1 - x^3$, so the product is $(1 - x^3)^6 (1 + x + x^2)$. The factor $(1 - x^3)^6$ has only the powers 0, 3, 6, 9, ..., so x^7 can only come from $x^6 \cdot x$: coefficient ${}^6C_2 \cdot 1 = 15$ (the x^6 term of $(1 - x^3)^6$ is $+{}^6C_2 x^6$). Expanding the two brackets separately would take dozens of terms.

Q11. Answer 14. From the first equation $w \neq 0$ and $z = w^{-5}$. Substituting in the second: $w^{-15} \cdot w = w^{-14} = i$, i.e. $w^{14} = -i$, which has exactly 14 distinct roots. Each fixes $z = w^{-5}$ uniquely, and every such pair satisfies both equations, so there are 14

pairs.

Q12. Answer 33. For $[[a, b], [c, d]]$ we need $ad = bc$, where each product is $-1, 0$ or 1 . Out of the 9 pairs (a, d) : 5 give product 0, 2 give 1 and 2 give -1 ; the same for (b, c) . The count is $5 \cdot 5 + 2 \cdot 2 + 2 \cdot 2 = 33$.

Q13. Answer 3. An AP of three numbers is fixed by its middle term and common difference $d \geq 1$: for each d the smallest term runs from 1 to $20 - 2d$, so there are $20 - 2d$ of them, $d = 1, \dots, 9$. Total $= \sum (20 - 2d) = 180 - 90 = 90$. $p = 90/2^9 C_3 = 90/1140 = 3/38$, so $38p = 3$.

Q14. Answer C. The roots are $a \pm \sqrt{3 - a}$, real only for $a \leq 3$. Let $f(x)$ be the left side. (P) Larger root < 3 : $\sqrt{3 - a} < 3 - a$, i.e. $3 - a > 1$, $a < 2$: $a = -5, \dots, 1$, seven values ($P \rightarrow 4$). (Q) Smaller root > 0 : $a > \sqrt{3 - a}$, i.e. $a > 0$ and $a^2 + a - 3 > 0$, $a > (\sqrt{13} - 1)/2 \approx 1.30$; with $a \leq 3$, $a = 2, 3$ ($Q \rightarrow 2$). (R) $f(-1) = a^2 + 3a - 2 < 0$: $-3.56 < a < 0.56$, so $a = -3, -2, -1, 0$ ($R \rightarrow 3$). (S) Need $f(-2) = a^2 + 5a + 1 > 0$, $f(2) = a^2 - 3a + 1 > 0$ and $-2 < a < 2$ for the vertex: this leaves $-0.21 < a < 0.38$, so $a = 0$ only (roots $\pm\sqrt{3}$) ($S \rightarrow 1$).

Q15. Answer A. (P) The general term ${}^{60}C_r 2^{(60-r)/2} 5^{r/3}$ is rational when $60 - r$ is even and r is a multiple of 3, i.e. $r = 0, 6, \dots, 60$: 11 terms ($P \rightarrow 3$). (Q) $2^5 = 32 \equiv -1 \pmod{33}$ and $2025 = 5 \cdot 405$, so $2^{2025} \equiv (-1)^{405} = -1 \equiv 32 \pmod{33}$ ($Q \rightarrow 4$). (R) Each factor gives a power $-2, -1, 1$ or 2 ; five of them give every integer from -10 to 10 (for example $0 = 1 + 1 + 1 - 1 - 2$), and all coefficients are positive, so nothing cancels: 21 terms ($R \rightarrow 1$). (S) ${}^{10}C_r/(r+1) = {}^{11}C_{r+1}/11$, so the sum is $(2^{11} - 1)/11 = 2047/11$ ($S \rightarrow 2$).

Q16. Answer D. (P) With $y_i = x_i - 1 \geq 0$, $y_1 + \dots + y_4 = 6$ with each $y_i \leq 4$: ${}^9C_3 = 84$ solutions without the cap, minus those with some $y_i \geq 5$ (4 choices of box $\times {}^4C_3 = 4$ ways to share the remaining 1): $84 - 16 = 68$ ($P \rightarrow 4$; 84 forgets the cap). (Q) Derangements of 5: $5!(1 - 1/1! + 1/2! - 1/3! + 1/4! - 1/5!) = 44$ ($Q \rightarrow 1$). (R) Any 4 distinct digits from 1–9 in increasing order (0 cannot appear, as it would have to lead): ${}^9C_4 = 126$ ($R \rightarrow 2$). (S) $7!/2! - 6! = 2520 - 720 = 1800$ ($S \rightarrow 3$).

Q17. Answer B. (P) Tosses 4 and 5 are H, toss 3 must be T, and tosses 1–2 must not be HH: 3 of the 2^5 sequences, $3/32$ ($P \rightarrow 2$). (Q) $P(\max \leq 4) - P(\max \leq 3) = (4^3 - 3^3)/6^3 = 37/216$ ($Q \rightarrow 3$). (R) Exactly 2 white in the first 4 draws, then white: $[{}^4C_2 \cdot {}^6C_2/{}^{10}C_4] \cdot 2/6 = (90/210)(1/3) = 1/7$ ($R \rightarrow 4$). (S) A wins on throw 1, 3, 5, ...: $(1/6)[1 + (25/36) + (25/36)^2 + \dots] = (1/6)(36/11) = 6/11$ ($S \rightarrow 1$).

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