



Calculus — JEE (Advanced) Pattern Sample Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (ADVANCED) PAPER 1

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	60	17 (4 + 3 + 6 + 4)	

GENERAL INSTRUCTIONS

1. This question paper has FOUR sections. All questions are compulsory.
2. Read the marking scheme of each section carefully: wrong answers in Sections 1, 2 and 4 carry negative marks.
3. Use of a calculator, log tables or any other aid is not allowed.
4. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

SECTION 1 (Q1–4) — +3 / 0 / –1

1. This section contains FOUR (04) questions.
2. Each question has FOUR options. ONLY ONE of these four options is the correct answer.
3. Full Marks: +3 if ONLY the correct option is chosen; Zero Marks: 0 if none of the options is chosen; Negative Marks: –1 in all other cases.

SECTION 2 (Q5–7) — +4 / partial / –2

1. This section contains THREE (03) questions.
2. Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).
3. Full Marks: +4 ONLY if (all) the correct option(s) is(are) chosen; Partial Marks: +3 if all the four options are correct but ONLY three options are chosen; +2 if three or more options are correct but ONLY two options are chosen, both of which are correct; +1 if two or more options are correct but ONLY one option is chosen and it is a correct option; Zero Marks: 0 if unanswered; Negative Marks: –2 in all other cases.

SECTION 3 (Q8–13) — +4 / 0

1. This section contains SIX (06) questions.
2. The answer to each question is a NON-NEGATIVE INTEGER.
3. Full Marks: +4 ONLY if the correct integer is entered; Zero Marks: 0 in all other cases.

SECTION 4 (Q14–17) — +3 / 0 / –1

1. This section contains FOUR (04) Matching List Sets.
2. Each set has ONE Multiple Choice Question. Each set has TWO lists: List-I and List-II. List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).
3. FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.
4. Full Marks: +3 ONLY if the option corresponding to the correct combination is chosen; Zero Marks: 0 if none of the options is chosen; Negative Marks: –1 in all other cases.

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Chapters covered: Limits · Continuity and Differentiability · Application of Derivatives · Definite Integration · Area under Curves · Differential Equations

SECTION 1 (Q1–4) · Single correct · +3 / 0 / –1

Each question has FOUR options. ONLY ONE of these four options is the correct answer.

1. The value of $\lim_{x \rightarrow 0} [(1+x)^{1/x} - e + ex/2] / x^2$ is
- (A) $e/24$
(B) $11e/24$
(C) $-11e/24$
(D) $11e/12$
2. The value of the integral $\int_0^1 \ln(1+x)/(1+x^2) dx$ is
- (A) $(\pi/4) \ln 2$
(B) $(\pi/2) \ln 2$
(C) $\pi \ln 2$
(D) $(\pi/8) \ln 2$
3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function such that $f(x+y) = f(x) + f(y) + 3xy(x+y)$ for all real x and y , and $f'(0) = -3$. The local maximum value of f is
- (A) 2
(B) -2
(C) 1
(D) 3
4. The solution curve of the differential equation $(x^2 - y^2) dx + 2xy dy = 0$ that passes through the point $(1, 1)$ is cut by the line $y = x$ into two regions. The area of the smaller region is
- (A) $\pi/2 - 1$
(B) $\pi/4 + 1/2$
(C) $\pi/4 - 1/2$
(D) $\pi - 2$

SECTION 2 (Q5–7) · One or more correct · +4 / partial / –2

Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).

5. Let $f(x) = \int_0^x (t^2 - 3t + 2) e^{-t} dt$ for all real x . Which of the following is/are correct?
- (A) The local maximum value of f is $1 - 1/e$.
(B) f has exactly one point of local minimum, and the local minimum value is $1 - 3/e^2$.
(C) $f(x) = 1$ for some $x > 0$.
(D) The equation $f(x) = 0$ has exactly one real root.
6. For integers $n \geq 2$, let $I_n = \int_0^{\pi/2} \sin^n x dx$. Which of the following is/are correct?
- (A) $I_6 = 5\pi/16$
(B) $n I_n = (n-1) I_{n-2}$
(C) $I_n I_{n+1} = \pi/(2(n+1))$
(D) $I_{n+1}/I_n \rightarrow 1$ as $n \rightarrow \infty$

7. Let $y = y(x)$ be the solution of $dy/dx = y + x$ with $y(0) = 0$. Which of the following is/are correct?

- (A) $y(x) \geq 0$ for all real x .
- (B) The area of the region bounded by the curve, the x -axis and the line $x = 1$ is $e - 5/2$.
- (C) The curve has a point of inflection.
- (D) The tangent to the curve at the point where $x = \ln 2$ meets the y -axis at $(0, 2 \ln 2 - 1)$.

SECTION 3 (Q8–13) · Non-negative integer · +4 / 0

The answer to each question is a NON-NEGATIVE INTEGER.

8. If $L = \lim_{(x \rightarrow 0)} [\tan(\sin x) - \sin(\tan x)] / x^7$, find the value of $1/L$.

Answer: _____

9. Let $I = \int_{-\pi}^{\pi} x \sin x / [(1 + 3^x)(1 + \cos^2 x)] dx$. Find the value of $8I/\pi^2$.

Answer: _____

10. Find the minimum value, for $x > 0$, of $[(x + 1/x)^6 - (x^6 + 1/x^6) - 2] / [(x + 1/x)^3 + (x^3 + 1/x^3)]$.

Answer: _____

11. Let $y = y(x)$, for $x < 1$, be the solution of $dy/dx + y = y^2 e^x$ with $y(0) = 1$. Find the value of $e \cdot [y(1/2)]^2$.

Answer: _____

12. Let $f(x) = (x^2 - 1)|x^2 - 3x + 2| + \cos|x| + (x - 2)|x^3 - 4x|$ for real x . Find the number of points at which f is not differentiable.

Answer: _____

13. Let $f(x) = x^3 + x + 1$, and let g be the inverse function of f . Find the value of $4 \int_1^3 g(y) dy$.

Answer: _____

SECTION 4 (Q14–17) · Matching lists · +3 / 0 / -1

Each set has ONE Multiple Choice Question. Each set has TWO lists: List-I and List-II. List-I has Four entries (P), (Q), (R) and (S) and List-II has Five entries (1), (2), (3), (4) and (5).

14. Match each limit in List-I with its value in List-II.

List-I	List-II
(P) $\lim_{(x \rightarrow 0)} (1 - \cos x \cdot \cos 2x \cdot \cos 3x)/x^2$	(1) $\pi/4$
(Q) $\lim_{(n \rightarrow \infty)} \sum n/(n^2 + k^2)$, over $k = 1$ to n	(2) $-e/2$
(R) $\lim_{(x \rightarrow 0)} (e^x - e^{\sin x})/(x - \sin x)$	(3) 7
(S) $\lim_{(x \rightarrow \infty)} x[(1 + 1/x)^x - e]$	(4) 1
	(5) $e/2$

- (A) $P \rightarrow 3, Q \rightarrow 1, R \rightarrow 4, S \rightarrow 5$
- (B) $P \rightarrow 1, Q \rightarrow 3, R \rightarrow 4, S \rightarrow 2$
- (C) $P \rightarrow 3, Q \rightarrow 1, R \rightarrow 4, S \rightarrow 2$
- (D) $P \rightarrow 3, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 2$

15. Match each integral in List-I with its value in List-II. ($[t]$ is the greatest integer less than or equal to t .)

List-I	List-II
(P) $\int_0^1 x \ln(1+x) dx$	(1) π
(Q) $\int_0^{\pi} x/(1+\sin x) dx$	(2) $1/4$
(R) $\int_0^{\infty} dx / [(1+x^2)(1+x^{2026})]$	(3) $\pi/4$
(S) $\int_0^2 [x^2] dx$	(4) $5 - \sqrt{2} - \sqrt{3}$
	(5) $3 - \sqrt{3}$

- (A) $P \rightarrow 2, Q \rightarrow 1, R \rightarrow 3, S \rightarrow 4$
 (B) $P \rightarrow 2, Q \rightarrow 3, R \rightarrow 1, S \rightarrow 4$
 (C) $P \rightarrow 2, Q \rightarrow 1, R \rightarrow 3, S \rightarrow 5$
 (D) $P \rightarrow 3, Q \rightarrow 1, R \rightarrow 2, S \rightarrow 4$

16. Match each entry in List-I with its value in List-II.

List-I	List-II
(P) The number of distinct real roots of $3x^4 - 4x^3 - 12x^2 + 5 = 0$	(1) $e^{1/e}$
(Q) The greatest value of $x^{1/x}$ for $x > 0$	(2) $e - 1$
(R) The least value of $2^{\sin x} + 2^{\cos x}$ for real x	(3) 3
(S) The value of c given by Lagrange's mean value theorem for $f(x) = \ln x$ on $[1, e]$	(4) $2^{1-1/\sqrt{2}}$
	(5) 2

- (A) $P \rightarrow 5, Q \rightarrow 1, R \rightarrow 4, S \rightarrow 2$
 (B) $P \rightarrow 3, Q \rightarrow 2, R \rightarrow 4, S \rightarrow 1$
 (C) $P \rightarrow 3, Q \rightarrow 1, R \rightarrow 2, S \rightarrow 4$
 (D) $P \rightarrow 3, Q \rightarrow 1, R \rightarrow 4, S \rightarrow 2$

17. Match each region in List-I with its area in List-II.

List-I	List-II
(P) The region bounded by $y = x^2 - 2 x $ and $y = 2 x $	(1) $16/3$
(Q) The region bounded by $y = e^x, y = e^{(-x)}$ and the line $x = \ln 2$	(2) $64/3$
(R) The region common to $y^2 \leq 4x$ and $x^2 \leq 4y$	(3) $2\sqrt{2}$
(S) The region between $y = \sin x$ and $y = \cos x$ for $\pi/4 \leq x \leq 5\pi/4$	(4) $1/2$
	(5) $32/3$

- (A) $P \rightarrow 5, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 3$
 (B) $P \rightarrow 2, Q \rightarrow 4, R \rightarrow 1, S \rightarrow 3$
 (C) $P \rightarrow 2, Q \rightarrow 4, R \rightarrow 3, S \rightarrow 1$
 (D) $P \rightarrow 1, Q \rightarrow 4, R \rightarrow 2, S \rightarrow 3$

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9
B	D	A	C	A, B, D	B, C, D	A, B	30	2

Q10	Q11	Q12	Q13	Q14	Q15	Q16	Q17
6	4	3	5	C	A	D	B

WORKED SOLUTIONS

Q1. Answer B. $(1+x)^{1/x} = \exp(\ln(1+x)/x) = \exp(1 - x/2 + x^2/3 - \dots) = e \cdot \exp(-x/2 + x^2/3 - \dots) = e[1 - x/2 + (1/3 + 1/8)x^2 + \dots] = e - ex/2 + (11e/24)x^2 + \dots$. So the numerator is $(11e/24)x^2 + O(x^3)$ and the limit is $11e/24$. The $x^2/8$ term comes from the square of $-x/2$ in the exponential series; it is the step most often dropped.

Q2. Answer D. Put $x = \tan \theta$: $I = \int_0^{\pi/4} \ln(1 + \tan \theta) d\theta$. Replace θ by $\pi/4 - \theta$: $1 + \tan(\pi/4 - \theta) = 1 + (1 - \tan \theta)/(1 + \tan \theta) = 2/(1 + \tan \theta)$. So $I = \int_0^{\pi/4} [\ln 2 - \ln(1 + \tan \theta)] d\theta = (\pi/4) \ln 2 - I$, giving $I = (\pi/8) \ln 2$. $(\pi/4) \ln 2$ is $2I$ — the step before halving.

Q3. Answer A. Put $x = y = 0$: $f(0) = 0$. Then $f'(x) = \lim_{h \rightarrow 0} [f(x+h) - f(x)]/h = \lim [f(h) + 3xh(x+h)]/h = f'(0) + 3x^2 = 3x^2 - 3$. With $f(0) = 0$, $f(x) = x^3 - 3x$ (and it does satisfy the equation, since $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$). $f' = 3(x-1)(x+1)$ changes from $+$ to $-$ at $x = -1$, so the local maximum value is $f(-1) = -1 + 3 = 2$; -2 is the local minimum value, at $x = 1$.

Q4. Answer C. Put $u = y^2$: the equation becomes $x du/dx - u + x^2 = 0$, i.e. $d(u/x)/dx = -1$, so $y^2/x = C - x$. Through $(1, 1)$, $C = 2$: $x^2 + y^2 = 2x$, the circle with centre $(1, 0)$ and radius 1. The line $y = x$ meets it at $(0, 0)$ and $(1, 1)$, whose radii from the centre, along $(-1, 0)$ and $(0, 1)$, are perpendicular. The smaller region is a quarter of the disc minus the right triangle with legs 1: $\pi/4 - 1/2$. $(\pi/2 - 1)$ is twice this.)

Q5. Answer A, B, D. For a polynomial p , $\int p(t)e^{-t} dt = -(p + p' + p'')e^{-t}$. Here $p + p' + p'' = t^2 - t + 1$, so $f(x) = 1 - (x^2 - x + 1)e^{-x}$. $f'(x) = (x-1)(x-2)e^{-x}$: $+$ on $(-\infty, 1)$, $-$ on $(1, 2)$, $+$ on $(2, \infty)$. (A) Local max $f(1) = 1 - 1/e$ — true. (B) The only local minimum is at $x = 2$, value $1 - 3/e^2$ — true. (C) $x^2 - x + 1 > 0$ always, so $f(x) < 1$ for every x — false (1 is only the limit as $x \rightarrow \infty$). (D) $f(0) = 0$; f increases on $(-\infty, 1)$, so $f < 0$ for $x < 0$; for $x > 0$, $f > 0$ since its least value there, $1 - 3/e^2 \approx 0.59$ at $x = 2$, is positive. Exactly one root, $x = 0$ — true.

Q6. Answer B, C, D. (B) Integrating by parts, $I_n = (n-1)(I_{n-2} - I_n)$, so $n I_n = (n-1)I_{n-2}$ — true. (A) $I_6 = (5/6)(3/4)(1/2)(\pi/2) = 5\pi/32$, not $5\pi/16$ — false. (C) By (B), $(n+1)I_{n+1}I_n = n I_n I_{n-1}$, so $(n+1)I_{n+1}$ is the same for every n : it equals $1 \cdot I_0 \cdot I_1 = \pi/2$ — true. (D) I_n decreases, so $I_{n+1} \leq I_n \leq I_{n-1} = (n+1)I_{n+1}/n$; dividing by I_n , $n/(n+1) \leq I_{n+1}/I_n \leq 1$ — true by squeezing.

Q7. Answer A, B. Linear equation: $(y e^{-x})' = x e^{-x}$, so $y e^{-x} = -(x+1)e^{-x} + C$; $y(0) = 0$ gives $C = 1$ and $y = e^x - x - 1$. (A) $e^x \geq 1 + x$ for all x — true. (B) $\int_0^1 (e^x - x - 1) dx = (e - 1) - 1/2 - 1 = e - 5/2$ — true. (C) $y'' = e^x > 0$, so no inflection — false. (D) At $x = \ln 2$: $y = 1 - \ln 2$ and slope $y' = e^x - 1 = 1$, so the tangent is $y = x + 1 - 2 \ln 2$, meeting the y -axis at $(0, 1 - 2 \ln 2)$, not $(0, 2 \ln 2 - 1)$ — false.

Q8. Answer 30. Both functions are odd, and composing the series of $\tan$ and $\sin$ gives $\tan(\sin x) = x + x^3/6 - x^5/40 - 107x^7/5040 + \dots$ and $\sin(\tan x) = x + x^3/6 - x^5/40 - 55x^7/1008 + \dots$. They agree up to x^5 — so any shortcut that stops at x^3 or x^5 (or uses L'Hôpital a few times) gives $0/0$ again. The difference is $(-107/5040 + 275/5040)x^7 = (168/5040)x^7 = x^7/30$. So $L = 1/30$ and $1/L = 30$.

Q9. Answer 2. $g(x) = x \sin x/(1 + \cos^2 x)$ is even. Replacing x by $-x$ turns $1/(1 + 3^x)$ into $3^x/(1 + 3^x)$; adding the two forms, $2I = \int$ from $-\pi$ to π of $g = 2 \int_0^\pi \pi x \sin x/(1 + \cos^2 x) dx$. Now replace x by $\pi - x$ and add: $I = (\pi/2) \int_0^\pi \pi \sin x/(1 + \cos^2 x) dx = (\pi/2)[- \tan^{-1}(\cos x)]_0^\pi = (\pi/2)(\pi/4 + \pi/4) = \pi^2/4$. So $8I/\pi^2 = 2$.

Q10. Answer 6. Let $t = x + 1/x \geq 2$. Then $x^3 + 1/x^3 = t^3 - 3t$ and $x^6 + 1/x^6 = (t^3 - 3t)^2 - 2$. The numerator is $t^6 - (t^3 - 3t)^2 = (3t)(2t^3 - 3t)$ and the denominator is $2t^3 - 3t$, so the expression equals $3t$, whose least value is 6, at $x = 1$. Differentiating the

original expression directly is hopeless.

Q11. Answer 4. Bernoulli equation: put $v = 1/y$, so $dv/dx = -y'/y^2$ and the equation becomes $dv/dx - v = -e^x$. Then $(v e^{(-x)})' = -1$, so $v e^{(-x)} = C - x$; $y(0) = 1$ gives $C = 1$. Hence $y = e^{(-x)}/(1 - x)$ and $y(1/2) = 2e^{(-1/2)}$, so $e \cdot y(1/2)^2 = e \cdot 4e^{(-1)} = 4$.

Q12. Answer 3. $\cos|x| = \cos x$ is differentiable everywhere. $(x^2 - 1)|x^2 - 3x + 2| = (x - 1)(x + 1)|x - 1||x - 2|$; at $x = 1$ the factor $(x - 1)|x - 1|$ is differentiable, so the only kink is at $x = 2$. $(x - 2)|x^3 - 4x| = (x - 2)|x - 2| \cdot |x||x + 2|$; differentiable at $x = 2$, with kinks at $x = 0$ and $x = -2$. The kink at 2 from the first term is not cancelled (the third term is smooth there), so f fails to be differentiable at $x = -2, 0$ and 2 : three points. Counting all five zeros ($\pm 2, 0, 1, 2$) of the moduli is the trap.

Q13. Answer 5. f is strictly increasing with $f(0) = 1$ and $f(1) = 3$. For an increasing f , $\int$ from $f(a)$ to $f(b)$ of $g(y) dy = b f(b) - a f(a) - \int_a^b f(x) dx$ (the two areas fill a rectangle minus a rectangle). With $a = 0, b = 1$: $\int_1^3 g = 3 - 0 - (1/4 + 1/2 + 1) = 5/4$, so the answer is 5. No formula for g is needed.

Q14. Answer C. (P) $\cos x \cos 2x \cos 3x = 1 - (1 + 4 + 9)x^2/2 + O(x^4)$, so the limit is 7 ($P \rightarrow 3$). (Q) It is the Riemann sum $(1/n) \sum 1/(1 + (k/n)^2) \rightarrow \int_0^1 dx/(1 + x^2) = \pi/4$ ($Q \rightarrow 1$). (R) $e^x - e^{(\sin x)} = e^{(\sin x)}(e^{(x - \sin x)} - 1)$, and $(e^u - 1)/u \rightarrow 1$ with $e^{(\sin x)} \rightarrow 1$: the limit is 1 ($R \rightarrow 4$). (S) $(1 + 1/x)^x = e \cdot \exp(-1/(2x) + O(1/x^2)) = e - e/(2x) + O(1/x^2)$, so the limit is $-e/2$ ($S \rightarrow 2$); $+e/2$ is the sign slip.

Q15. Answer A. (P) By parts: $(x^2/2) \ln(1 + x)$ from 0 to 1, minus $\frac{1}{2} \int_0^1 x^2/(1 + x) dx = (\ln 2)/2 - \frac{1}{2}(1/2 - 1 + \ln 2) = 1/4$ ($P \rightarrow 2$). (Q) Replace x by $\pi - x$ and add: $I = (\pi/2) \int_0^\pi dx/(1 + \sin x) = (\pi/2) \cdot 2 = \pi$ ($Q \rightarrow 1$). (R) Put $x = 1/t$: $I = \int_0^\infty t^{2026} dt/[(1 + t^2)(1 + t^{2026})]$; adding, $2I = \int_0^\infty dx/(1 + x^2) = \pi/2$, so $I = \pi/4$ whatever the power ($R \rightarrow 3$). (S) $[x^2]$ is 0, 1, 2, 3 on $[0, 1]$, $[1, \sqrt{2}]$, $[\sqrt{2}, \sqrt{3}]$, $[\sqrt{3}, 2]$: $(\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3}) = 5 - \sqrt{2} - \sqrt{3}$ ($S \rightarrow 4$).

Q16. Answer D. (P) $f'(x) = 12x(x + 1)(x - 2)$. $f(-1) = 3 + 4 - 12 + 5 = 0$: the local minimum at $x = -1$ touches the axis, a double root. $f(0) = 5 > 0$, $f(2) = -27 < 0$ and $f \rightarrow \infty$, so there is one more root in $(0, 2)$ and one in $(2, \infty)$: 3 distinct real roots ($P \rightarrow 3$); counting sign changes alone misses the touching root. (Q) $\ln y = (\ln x)/x$ has derivative $(1 - \ln x)/x^2$, greatest at $x = e$: $e^{(1/e)}$ ($Q \rightarrow 1$). (R) $AM \geq GM$: $2^{(\sin x)} + 2^{(\cos x)} \geq 2 \cdot 2^{((\sin x + \cos x)/2)} \geq 2 \cdot 2^{(-1/\sqrt{2})} = 2^{(1 - 1/\sqrt{2})}$, with equality at $x = 5\pi/4$ ($R \rightarrow 4$). (S) $1/c = (1 - 0)/(e - 1)$, so $c = e - 1$ ($S \rightarrow 2$).

Q17. Answer B. (P) The curves meet where $x^2 - 2|x| = 2|x|$, i.e. $x = 0, \pm 4$; by symmetry the area is $2 \int_0^4 (2x - x^2 + 2x) dx = 2(32 - 64/3) = 64/3$ ($P \rightarrow 2$); $32/3$ is only one half. (Q) $\int_0^1 (\ln 2) (e^x - e^{(-x)}) dx = [e^x + e^{(-x)}]_0^1 (\ln 2) = 5/2 - 2 = 1/2$ ($Q \rightarrow 4$). (R) The parabolas meet at $(0, 0)$ and $(4, 4)$: $\int_0^4 (2\sqrt{x} - x^2/4) dx = 32/3 - 16/3 = 16/3$ ($R \rightarrow 1$). (S) $\sin x \geq \cos x$ on $[\pi/4, 5\pi/4]$: $[-\cos x - \sin x] = \sqrt{2} - (-\sqrt{2}) = 2\sqrt{2}$ ($S \rightarrow 3$).

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