



Continuity and Differentiability — JEE (Main) Pattern Sample Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, -1 for an incorrect answer, 0 if the question is not attempted.
5. $[x]$ denotes the greatest integer $\leq x$ and $\log$ denotes the natural logarithm. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Continuity · Greatest integer function · Differentiability · Differentiability of composite functions · Derivative as a limit · Chain rule · Derivatives of inverse trigonometric functions · Derivative of an inverse function · Implicit differentiation · Parametric differentiation · Logarithmic differentiation · Derivative of one function with respect to another · Second order derivatives

SECTION A (Q1–20) · Multiple choice · +4 / 0 / –1

Each question has FOUR options. ONLY ONE of them is correct.

1. The function

$$f(x) = (e^{ax} - e^x - x)/x^2, x \neq 0$$

$$f(x) = b, x = 0$$

is continuous at $x = 0$. Then $a + 2b$ is equal to

- (1) 3
- (2) 4
- (3) 5
- (4) 6

2. Let $f(x) = (x^2 - 2)[x^2]$, $x \in (-2, 2)$, where $[t]$ is the greatest integer $\leq t$. If f is discontinuous at m points, and continuous but not differentiable at n points, of $(-2, 2)$, then $m + n$ is equal to

- (1) 4
- (2) 5
- (3) 6
- (4) 8

3. The number of points at which $f(x) = (x + 2)|x^2 - 4| - 8|x^2 - 2x|$, $x \in \mathbb{R}$, is NOT differentiable is

- (1) 0
- (2) 1
- (3) 2
- (4) 3

4. Let $g(x)$ be the derivative of $\sin^{-1}(2x/(1 + x^2))$ with respect to $\cos^{-1}((1 - x^2)/(1 + x^2))$, wherever both are differentiable. Then the values of $g(2)$ and $g(-1/2)$ are

- (1) 1 and 1
- (2) -1 and 1
- (3) 1 and -1
- (4) -1 and -1

5. If $x = 2 \cos t - \cos 2t$ and $y = 2 \sin t - \sin 2t$, then at $t = 2\pi/3$ the values of dy/dx and d^2y/dx^2 are respectively

- (1) 0 and $-\sqrt{3}/4$
- (2) 0 and $\sqrt{3}/4$
- (3) $\sqrt{3}$ and $-\sqrt{3}/4$
- (4) 0 and $-3/4$

6. If $f(x) = x^{(\sin(\pi x/2))} + (\sin(\pi x/6))^x$, $x \in (0, 3)$, then $f'(1)$ is equal to

- (1) $1 + (1/2) \log 2 + \pi/(4\sqrt{3})$
- (2) $(1/2)(1 - \log 2) + \pi/(4\sqrt{3})$
- (3) $1 - (1/2) \log 2 + \pi\sqrt{3}/4$
- (4) $1 - (1/2) \log 2 + \pi/(4\sqrt{3})$

7. Let f be a differentiable function with $f(1) = 1$ and $f'(1) = 3$. If $g(x) = f(x \cdot f(x^2))$, then $g'(1)$ is equal to

- (1) 9
- (2) 12
- (3) 21
- (4) 27

8. Let $f(x) = |x - 1|$ and $g(x) = x^2 - 2|x|$, $x \in \mathbb{R}$. If $f \circ g$ is not differentiable at m points and $g \circ f$ is not differentiable at n points, then $m + n$ is equal to

- (1) 3
- (2) 4
- (3) 5
- (4) 6

9. Let

$$f(x) = a \sin x + b \cos x + c, x < 0$$

$$f(x) = e^{2x}, x \geq 0.$$

If f is twice differentiable at $x = 0$, then (a, b, c) is

- (1) (2, -4, 5)
- (2) (2, 4, -3)
- (3) (1, -1, 2)
- (4) (2, -4, -3)

10. Let $y = e^{a \tan^{-1} x}$, where a is a non-zero real number. If $d^2y/dx^2 = 0$ at $x = \sqrt{3}$, then a is equal to

- (1) $\sqrt{3}$
- (2) $2\sqrt{3}$
- (3) $4\sqrt{3}$
- (4) $2/\sqrt{3}$

11. Given below are two statements.

Statement I: If f is differentiable at $x = a$ and g is not differentiable at $x = a$, then the product $f \cdot g$ cannot be differentiable at $x = a$.

Statement II: If $f + g$ and f are both differentiable at $x = a$, then g is differentiable at $x = a$.

In the light of the above statements, choose the correct answer from the options given below:

- (1) Both Statement I and Statement II are true
- (2) Both Statement I and Statement II are false
- (3) Statement I is true but Statement II is false
- (4) Statement I is false but Statement II is true

12. Let f be a differentiable function with $f(1) = 2$ and $f'(1) = 3$ (and $f(x) > 0$ near $x = 1$). Then $\lim_{x \rightarrow 1} [f(x)/f(1)]^{1/(\sqrt{x} - 1)}$ is equal to

- (1) $e^{3/2}$
- (2) $e^{3/4}$
- (3) e^3
- (4) e^6

13. Let $f(x) = \sin^{-1}(\sin x) + \cos^{-1}(\cos x)$, $x \in (0, 2\pi)$. The number of points in $(0, 2\pi)$ at which f is not differentiable, and the value of $f'(5\pi/4)$, are respectively
- 4 and 2
 - 3 and 0
 - 2 and -2
 - 3 and -2
14. Let $f(x) = x^5 + 3x^3 + 2x - 2$, $x \in \mathbb{R}$, and let g be the inverse function of f . Then $g''(4)$ is equal to
- $-1/16$
 - $-19/2048$
 - $19/2048$
 - $-19/128$
15. The derivative of $x^2 e^x$ with respect to $x e^{(x^2)}$ at $x = 2$ is
- $9/(8e^2)$
 - $8e^2/9$
 - $4/(9e^2)$
 - $8/(9e^2)$
16. Let $f(x) = \lim_{n \rightarrow \infty} [ax^2 + bx + 2 + x^{(2n)} \cdot x^3]/(1 + x^{(2n)})$, $x \in \mathbb{R}$. If f is continuous on $\mathbb{R}$, then
- $a = -2$, $b = 1$, and f is differentiable on $\mathbb{R}$
 - $a = -2$, $b = 1$, and f is not differentiable at $x = 1$ and at $x = -1$
 - $a = -1$, $b = 0$, and f is differentiable on $\mathbb{R}$
 - $a = -2$, $b = 1$, and f is not differentiable at $x = 1$ only
17. Which of the following functions is differentiable at $x = 0$? ($[t]$ denotes the greatest integer $\leq t$.)
- $[x] \sin(\pi x)$
 - $\sin|x|$
 - $|x| \sin x$
 - $\cos^{-1}(\cos x)$
18. If the function
- $$f(x) = [\cos(\tan x) - \cos(\sin x)]/x^4, x \neq 0$$
- $$f(x) = k, x = 0$$
- is continuous at $x = 0$, then k is equal to
- $-1/2$
 - $1/2$
 - $-1/6$
 - $-1/4$
19. Let $f(x) = [x] + 3(x - [x])^2 - 2(x - [x])^3$, $x \in \mathbb{R}$, where $[t]$ is the greatest integer $\leq t$. Then f is
- continuous and differentiable at every real x
 - discontinuous at every integer
 - continuous at every real x , but not differentiable at any integer
 - continuous at every real x , and differentiable everywhere except at $x = 0$

20. Let $f(x) = x^3 - 3x^2 + 4$. The function $|f(x)|$ is not differentiable at

- (1) $x = -1$ only
- (2) $x = 2$ only
- (3) $x = -1$ and $x = 2$
- (4) no real x

SECTION B (Q21–25) · Numerical value · +4 / 0 / –1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

21. Let $y = y(x)$ be the differentiable function defined near $x = 1$ by $x^2 + xy + y^3 = 3$ with $y(1) = 1$. Then the value of $-32 \cdot \frac{d^2y}{dx^2}$ at $x = 1$ is _____.

Answer: _____

22. Let $f(x) = (1 + x)(1 + 2x)(1 + 3x) \dots (1 + 10x)$. Then the value of $f''(0)/f'(0)$ is _____.

Answer: _____

23. Let $f(x) = [x^2] - [2x]$, $x \in (0, 3)$, where $[t]$ is the greatest integer $\leq t$. The number of points of $(0, 3)$ at which f is discontinuous is _____.

Answer: _____

24. Let $f(x) = x^3 + x$, $x \in \mathbb{R}$, let g be the inverse function of f , and let $h(x) = g(g(x))$. Then the value of $1/h'(10)$ is _____.

Answer: _____

25. Let $f(x) = x^3 - 6x^2 + 9x$ and $g(x) = \max \{f(t) : 0 \leq t \leq x\}$, $0 \leq x \leq 5$. The function g is not differentiable at exactly one point $x = \alpha$ of $(0, 5)$. The value of $\alpha + g(\alpha) + g'(5)$ is _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(3)	(3)	(2)	(4)	(1)	(4)	(3)	(2)	(1)	(2)
Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(4)	(3)	(4)	(2)	(4)	(2)	(3)	(1)	(1)	(1)
Q21		Q22		Q23		Q24		Q25	
31		48		9		52		32	

WORKED SOLUTIONS

Q1. Answer (3). $e^{ax} - e^x - x = (a - 1 - 1)x + (a^2 - 1)x^2/2 + O(x^3)$. For the limit as $x \rightarrow 0$ to be finite the x term must vanish, so $a = 2$, and then the limit is $(a^2 - 1)/2 = 3/2$. Continuity gives $b = 3/2$, so $a + 2b = 2 + 3 = 5$.

Q2. Answer (3). $[x^2]$ jumps only where $x^2 = 1, 2$ or 3 , i.e. at $x = \pm 1, \pm\sqrt{2}, \pm\sqrt{3}$ (near $x = 0$ it is 0). At $x = \pm 1$ and $\pm\sqrt{3}$ the factor $x^2 - 2$ is -1 and 1 , not 0 , so f jumps: $m = 4$. At $x = \pm\sqrt{2}$ the factor is 0 , so f is continuous; but just left of $\sqrt{2}$, $f = x^2 - 2$ with slope $2\sqrt{2}$, and just right, $f = 2(x^2 - 2)$ with slope $4\sqrt{2}$, so f is not differentiable there (similarly at $-\sqrt{2}$): $n = 2$. $m + n = 6$.

Q3. Answer (2). $(x + 2)|x^2 - 4| = (x + 2)|x + 2| \cdot |x - 2|$. $(x + 2)|x + 2|$ is differentiable (also at -2), so the first term can fail only at $x = 2$, where it behaves like $16|x - 2|$ (corner: slopes jump by $2 \times 16 = 32$). The second term is $-8|x||x - 2|$: corners at $x = 0$ and at $x = 2$, where it behaves like $-16|x - 2|$ (slopes jump by -32). At $x = 2$ the two jumps cancel, so f is differentiable there; only $x = 0$ remains. Answer 1, not 2.

Q4. Answer (4). Put $x = \tan \theta$, $\theta \in (-\pi/2, \pi/2)$. $2x/(1 + x^2) = \sin 2\theta$ and $(1 - x^2)/(1 + x^2) = \cos 2\theta$. For $x > 1$, $2\theta \in (\pi/2, \pi)$: $\sin^{-1}(\sin 2\theta) = \pi - 2\theta$, $\cos^{-1}(\cos 2\theta) = 2\theta$, so $g = -1$. For $-1 < x < 0$, $2\theta \in (-\pi/2, 0)$: $\sin^{-1}(\sin 2\theta) = 2\theta$, $\cos^{-1}(\cos 2\theta) = -2\theta$, so $g = -1$ again. Hence $g(2) = -1$ and $g(-1/2) = -1$. Taking both as $2 \tan^{-1}x$ everywhere gives 1 and 1.

Q5. Answer (1). $dy/dx = (2 \cos t - 2 \cos 2t)/(2 \sin 2t - 2 \sin t) = [2 \sin(3t/2) \sin(t/2)]/[2 \cos(3t/2) \sin(t/2)] = \tan(3t/2)$, which is $\tan \pi = 0$ at $t = 2\pi/3$. $d^2y/dx^2 = [d(\tan(3t/2))/dt] \div (dx/dt) = (3/2) \sec^2(3t/2) \div (2 \sin 2t - 2 \sin t)$. At $t = 2\pi/3$: $\sec^2 \pi = 1$ and $2 \sin(4\pi/3) - 2 \sin(2\pi/3) = -\sqrt{3} - \sqrt{3} = -2\sqrt{3}$, so $d^2y/dx^2 = (3/2)/(-2\sqrt{3}) = -\sqrt{3}/4$.

Q6. Answer (4). $u = x^{(\sin(\pi x/2))}$: $\log u = \sin(\pi x/2) \log x$, so $u' = u[(\pi/2) \cos(\pi x/2) \log x + \sin(\pi x/2)/x]$; at $x = 1$, $u = 1$ and $u' = 0 + 1 = 1$. $v = (\sin(\pi x/6))^x$: $\log v = x \log \sin(\pi x/6)$, so $v' = v[\log \sin(\pi x/6) + x(\pi/6) \cot(\pi x/6)]$; at $x = 1$, $v = 1/2$ and $v' = (1/2)[- \log 2 + (\pi/6)\sqrt{3}] = -(1/2) \log 2 + \pi\sqrt{3}/12$. $\pi\sqrt{3}/12 = \pi/(4\sqrt{3})$, so $f'(1) = 1 - (1/2) \log 2 + \pi/(4\sqrt{3})$.

Q7. Answer (3). $g'(x) = f'(x f(x^2)) \cdot [f(x^2) + x \cdot f'(x^2) \cdot 2x]$. At $x = 1$: $x f(x^2) = 1$, so $g'(1) = f'(1) \cdot [f(1) + 2f'(1)] = 3 \times (1 + 6) = 21$. Dropping the factor $2x$ from the derivative of x^2 gives 12.

Q8. Answer (2). $(f \circ g)(x) = |x^2 - 2|x|| - 1$. Inside the modulus, $|x|^2 - 2|x| - 1 = 0$ only for $|x| = 1 + \sqrt{2}$, where it changes sign: corners at $x = \pm(1 + \sqrt{2})$. At $x = 0$ the inside is -1 , so $f \circ g = 1 + 2|x| - x^2$ there: a corner. So $m = 3$. $(g \circ f)(x) = (x - 1)^2 - 2|x - 1|$: a corner at $x = 1$ only (slopes 2 and -2), so $n = 1$. $m + n = 4$.

Q9. Answer (1). Continuity: $b + c = e^0 = 1$. First derivative: $a \cos x - b \sin x \rightarrow a$ from the left, $2e^{(2x)} \rightarrow 2$ from the right, so $a = 2$. Second derivative: $-a \sin x - b \cos x \rightarrow -b$ from the left, $4e^{(2x)} \rightarrow 4$ from the right, so $b = -4$ and $c = 5$. $(2, 4, -3)$ takes the second derivative of $b \cos x$ as $+b \cos x$; $(1, -1, 2)$ forgets the factor 2 from $e^{(2x)}$.

Q10. Answer (2). $y' = a y/(1 + x^2)$, i.e. $(1 + x^2)y' = ay$. Differentiating, $(1 + x^2)y'' + 2x y' = a y'$, so $y'' = (a - 2x) y'/(1 + x^2)$. Since $y' = ay/(1 + x^2) \neq 0$ for $a \neq 0$, $y'' = 0$ exactly where $x = a/2$. So $a = 2\sqrt{3}$.

Q11. Answer (4). Statement I is false: $f(x) = x$ is differentiable and $g(x) = |x|$ is not differentiable at 0, but $f \cdot g = x|x|$ is differentiable at 0 (its difference quotient $h|h|/h = |h| \rightarrow 0$). Statement II is true: $g = (f + g) - f$ is the difference of two functions differentiable at a , so it is differentiable at a .

- Q12. Answer (3).** The limit is of the form 1^∞ . Its logarithm is $\lim [\log f(x) - \log f(1)]/(\sqrt{x} - 1) = \lim \{[\log f(x) - \log f(1)]/(x - 1)\} \cdot \{(x - 1)/(\sqrt{x} - 1)\} = [f'(1)/f(1)] \cdot \lim (\sqrt{x} + 1) = (3/2) \times 2 = 3$. So the limit is e^3 . $e^{(3/2)}$ forgets the factor $\sqrt{x} + 1 \rightarrow 2$.
- Q13. Answer (4).** On $(0, \pi/2)$: $f = x + x = 2x$. On $(\pi/2, \pi)$: $f = (\pi - x) + x = \pi$. On $(\pi, 3\pi/2)$: $f = (\pi - x) + (2\pi - x) = 3\pi - 2x$. On $(3\pi/2, 2\pi)$: $f = (x - 2\pi) + (2\pi - x) = 0$. The slopes 2, 0, -2, 0 change at $\pi/2$, π and $3\pi/2$: 3 points. At $5\pi/4$, $f' = -2$.
- Q14. Answer (2).** $f'(x) = 5x^4 + 9x^2 + 2 > 0$, so f is invertible; $f(1) = 4$, so $g(4) = 1$. From $f(g(y)) = y$: $g' = 1/f'(g)$, and differentiating again, $g'' = -f''(g) g'/f'(g)^2 = -f''(g)/f'(g)^3$. $f'(1) = 16$ and $f''(x) = 20x^3 + 18x$ gives $f''(1) = 38$. $g''(4) = -38/16^3 = -38/4096 = -19/2048$. $-19/128$ uses $f'(1)^2$ instead of $f'(1)^3$; $-1/16$ is $g'(4)$ with a minus sign.
- Q15. Answer (4).** $d(x^2 e^x)/dx = (x^2 + 2x)e^x$, which is $8e^2$ at $x = 2$. $d(x e^{(x^2)})/dx = (1 + 2x^2)e^{(x^2)}$, which is $9e^4$ at $x = 2$. The required derivative is the ratio, $8e^2/9e^4 = 8/(9e^2)$. (At $x = 1$ the two derivatives are equal, $3e$ each, but not at $x = 2$.)
- Q16. Answer (2).** For $|x| < 1$, $x^{(2n)} \rightarrow 0$ and $f(x) = ax^2 + bx + 2$; for $|x| > 1$, dividing by $x^{(2n)}$, $f(x) = x^3$; $f(1) = (a + b + 3)/2$ and $f(-1) = (a - b + 1)/2$. Continuity at 1: $a + b + 2 = 1$; at -1: $a - b + 2 = -1$. So $a = -2$, $b = 1$ (and then $f(\pm 1) = \pm 1$). At $x = 1$ the slopes are $2a + b = -3$ from the left and 3 from the right; at $x = -1$ they are 3 (from x^3) on the left and $-2a + 1 + b = 5$ on the right. So f is not differentiable at either point. ($a = -1$, $b = 0$ makes f continuous at 1 but not at -1.)
- Q17. Answer (3).** (1) For $-1 < x < 0$, $[x] \sin \pi x = -\sin \pi x$ (slope $-\pi$ at 0); for $0 \leq x < 1$ it is 0: not differentiable. (2) $\sin|x|$ has slopes -1 and 1 at 0. (4) $\cos^{-1}(\cos x) = |x|$ for $|x| \leq \pi$: not differentiable. (3) $[|h| \sin h - 0]/h = |h| (\sin h/h) \rightarrow 0$ from both sides, so $|x| \sin x$ is differentiable at 0 with derivative 0.
- Q18. Answer (1).** $\cos u = 1 - u^2/2 + u^4/24 - \dots$. $\tan^2 x = x^2 + (2/3)x^4 + \dots$ and $\sin^2 x = x^2 - (1/3)x^4 + \dots$, while $\tan^4 x$ and $\sin^4 x$ both start with x^4 . So $\cos(\tan x) - \cos(\sin x) = -(1/2)(\tan^2 x - \sin^2 x) + O(x^6) = -(1/2)(x^4) + O(x^6)$, and $k = -1/2$.
- Q19. Answer (1).** Write $u = x - [x] \in [0, 1)$. Between integers $f = [x] + 3u^2 - 2u^3$ is a polynomial in x . At an integer n : as $x \rightarrow n^-$, $[x] = n - 1$ and $u \rightarrow 1$, so $f \rightarrow n - 1 + 3 - 2 = n$; as $x \rightarrow n^+$, $f \rightarrow n = f(n)$. So f is continuous everywhere. Its slope between integers is $6u - 6u^2 = 6u(1 - u)$, which tends to 0 both as $u \rightarrow 1$ (from the left of n) and as $u \rightarrow 0$ (from the right), so $f'(n) = 0$ exists at every integer too. f is differentiable everywhere, although $[x]$ alone jumps.
- Q20. Answer (1).** $f(x) = (x + 1)(x - 2)^2$. $|f|$ can fail to be differentiable only where $f = 0$. At $x = -1$ (a simple root) f changes sign with $f'(-1) = 9 \neq 0$, so $|f|$ has slopes -9 and 9: not differentiable. At $x = 2$ (a double root) $f \geq 0$ nearby, so $|f| = f$ there, which is differentiable. Answer: $x = -1$ only.
- Q21. Answer 31.** Differentiating: $2x + y + xy' + 3y^2y' = 0$; at $(1, 1)$, $3 + 4y' = 0$, so $y' = -3/4$. Differentiating again: $2 + 2y' + xy'' + 6y(y')^2 + 3y^2y'' = 0$; at $(1, 1)$: $2 - 3/2 + 6(9/16) + 4y'' = 0$, i.e. $4y'' = -31/8$, $y'' = -31/32$. So $-32y'' = 31$.
- Q22. Answer 48.** Near 0, $f(x) = 1 + (\Sigma k)x + (\Sigma_{\{i < j\}} ij)x^2 + \dots$, so $f'(0) = \Sigma k = 55$ and $f''(0) = 2\Sigma_{\{i < j\}} ij = (\Sigma k)^2 - \Sigma k^2 = 3025 - 385 = 2640$. (Equivalently, from $f'/f = \Sigma k/(1 + kx)$: $f''/f - (f'/f)^2 = -\Sigma k^2$.) $f''(0)/f(0) = 2640/55 = 48$.
- Q23. Answer 9.** $[x^2]$ jumps by 1 where $x^2 = 1, 2, \dots, 8$, i.e. at $x = 1, \sqrt{2}, \sqrt{3}, 2, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}$; $[2x]$ jumps by 1 at $x = 1/2, 1, 3/2, 2, 5/2$. At $x = 1$ and $x = 2$ both jump up by 1 at the same point, so the jumps cancel and f is continuous there (e.g. $f = 0 - 1$ just left of 1 and $1 - 2$ just right of it). The other $6 + 3 = 9$ points remain: $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}, \sqrt{7}, \sqrt{8}, 1/2, 3/2$ and $5/2$.
- Q24. Answer 52.** $f(2) = 10$ and $f(1) = 2$, so $g(10) = 2$ and $g(2) = 1$. $h'(10) = g'(g(10)) \cdot g'(10) = g'(2) \cdot g'(10) = [1/f'(1)] \cdot [1/f'(2)]$ with $f'(x) = 3x^2 + 1$: $f'(1) = 4$ and $f'(2) = 13$. So $h'(10) = 1/52$ and $1/h'(10) = 52$.
- Q25. Answer 32.** $f' = 3(x - 1)(x - 3)$: f rises to $f(1) = 4$, falls to $f(3) = 0$ and rises again. So $g = f$ on $[0, 1]$, $g = 4$ from $x = 1$ until f climbs back to 4, and $g = f$ after that. $f(x) = 4$ gives $(x - 1)^2(x - 4) = 0$, so that happens at $x = 4$. At $x = 1$, $f'(1) = 0$, so g is differentiable; at $x = 4$ the slopes are 0 and $f'(4) = 9$: $\alpha = 4$ and $g(\alpha) = 4$. $g'(5) = f'(5) = 75 - 60 + 9 = 24$. Total $4 + 4 + 24 = 32$.