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Binomial Theorem — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: General term · Middle term · Binomial coefficients · Applications of the binomial theorem

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. The term independent of x in the expansion of $(2x^2 - 1/x)^9$ is

(1) 84

(2) −672

(3) 672

(4) 336

2. The coefficient of the middle term in the expansion of $(1 - 3x)^{10}$ is

(1) −61236

(2) 61236

(3) −20412

(4) −30618

3. The coefficient of x^6 in the expansion of $(1 + x)^{10}(1 - x)^{10}$ is

(1) 120

(2) 210

(3) 0

(4) −120

4. If $C_0, C_1, \ldots, C_{10}$ are the binomial coefficients in the expansion of $(1 + x)^{10}$, then $C_0 + 2C_1 + 3C_2 + \ldots + 11C_{10}$ is equal to

(1) 5120

(2) 6144

(3) 1024

(4) 6656

5. The remainder when 7^{103} is divided by 25 is

(1) 24

(2) 7

(3) 13

(4) 18

6. The number of non-zero terms in the expansion of $(x + 2y)^7 + (x - 2y)^7$, after simplification, is

(1) 4

(2) 8

(3) 7

(4) 3

7. If the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are equal, then a is equal to

(1) 7/9

(2) 3

(3) 9/7

(4) 27/7

8. The value of $(\sqrt{2} + 1)^6 + (\sqrt{2} - 1)^6$ is

(1) 196

(2) $140\sqrt{2}$

(3) 99

(4) 198

9. Statement I: $9^n - 8n - 1$ is divisible by 64 for every positive integer n.
Statement II: $8^n - 7n - 1$ is divisible by 49 for every positive integer n.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

10. The coefficient of x^4 in $(1 + x)^4 + (1 + x)^5 + (1 + x)^6 + \ldots + (1 + x)^{10}$ is

(1) 330

(2) 462

(3) 792

(4) 210

11. In the expansion of $(ax + 1/x)^n$, $a > 0$, the 4th term is independent of x and is equal to 5/2. Then (a, n) is

(1) (1, 6)

(2) (1/2, 3)

(3) (1/2, 6)

(4) (2, 6)

12. Which of the following is true?

(1) $99^{50} + 100^{50} = 101^{50}$

(2) $101^{50} < 100^{50}$

(3) $99^{50} + 100^{50} > 101^{50}$

(4) $101^{50} > 99^{50} + 100^{50}$

13. If $C_0, C_1, \ldots, C_{10}$ are the binomial coefficients in the expansion of $(1 + x)^{10}$, then $C_0C_1 + C_1C_2 + C_2C_3 + \ldots + C_9C_{10}$ is equal to

(1) 184756

(2) 167960

(3) 125970

(4) 92378

14. The coefficient of x^4 in the expansion of $(x/2 - 3/x^2)^{10}$ is

(1) −405/256

(2) 135/256

(3) 405/256

(4) 405/128

15. The sum of the coefficients in the expansion of $(1 + 2x)^n$ is 6561. The greatest coefficient in this expansion is

(1) 1792

(2) 1024

(3) 1120

(4) 448

16. Assertion A: In the expansion of $(1 + x)^{10}$, the coefficient of x^5 is equal to the coefficient of x^6.
Reason R: ${}^nC_r = {}^nC_{n-r}$ for $0 \le r \le n$.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

17. The coefficient of x^3 in the expansion of $(1 - x + x^2)(1 + x)^{10}$ is

(1) 75

(2) 85

(3) 165

(4) 120

18. In the expansion of $(1 + x)^n$ the coefficients of the 3rd and the 13th terms are equal. The coefficient of the middle term is

(1) 3432

(2) 1716

(3) 3003

(4) 6435

19. For every positive integer n, the ratio of the coefficient of x^n in $(1 + x)^{2n}$ to the coefficient of x^n in $(1 + x)^{2n-1}$ is

(1) 1

(2) 1/2

(3) 2

(4) n

20. The coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1 : 7 : 42. Then n is equal to

(1) 48

(2) 55

(3) 56

(4) 63

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

21. The coefficient of x^{10} in the expansion of $(1 + x^2)^5 (1 + x)^4$ is _____.

Answer: _____

22. The remainder when 11^{20} is divided by 100 is _____.

Answer: _____

23. The value of $1000 \times (1.002)^{10}$, correct to the nearest integer, is _____.

Answer: _____

24. The number of values of r, $0 \le r \le 20$, for which ${}^{20}C_r > 10000$ is _____.

Answer: _____

25. The coefficient of x^5 in the expansion of $(1 + 2x)^6 (1 - x)^7$ is _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(3)	(1)	(4)	(2)	(4)	(1)	(3)	(4)	(1)	(2)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(3)	(4)	(2)	(3)	(1)	(4)	(2)	(1)	(3)	(2)

Q21	Q22	Q23	Q24	Q25
41	61	1020	11	171

WORKED SOLUTIONS

Q1. Answer (3). $T_{r+1} = {}^{9}C_r\,(2x^2)^{9-r}\,(-1/x)^r = {}^{9}C_r\,2^{9-r}\,(-1)^r\,x^{18-3r}$. The power is 0 for r = 6: ${}^{9}C_6 \times 2^3 \times 1 = 84 \times 8 = 672$.

Q2. Answer (1). There are 11 terms, so the middle one is $T_6 = {}^{10}C_5\,(-3x)^5$, with coefficient $252 \times (-243) = -61236$.

Q3. Answer (4). $(1 + x)^{10}(1 - x)^{10} = (1 - x^2)^{10}$. The term in $x^6 = (x^2)^3$ is ${}^{10}C_3\,(-x^2)^3$, with coefficient −120.

Q4. Answer (2). $\Sigma\,(r + 1){}^{n}C_r = \Sigma\,r{}^{n}C_r + \Sigma\,{}^{n}C_r = 10 \times 2^9 + 2^{10} = 5120 + 1024 = 6144$. (${}^{n}C_r = 10 \times {}^{9}C_{r-1}$, which adds up to 10×2^9.)

Q5. Answer (4). $7^2 = 49 = 50 - 1$, so $7^{103} = 7 \times (50 - 1)^{51}$. Every term of $(50 - 1)^{51}$ except the last contains 50, so $(50 - 1)^{51} \equiv -1 \pmod{25}$ and $7^{103} \equiv -7 \equiv 18 \pmod{25}$.

Q6. Answer (1). The terms with odd powers of 2y cancel and those with even powers double: $2[x^7 + {}^{7}C_2\,x^5(2y)^2 + {}^{7}C_4\,x^3(2y)^4 + {}^{7}C_6\,x(2y)^6]$. That leaves 4 terms.

Q7. Answer (3). Coefficient of x^2: ${}^{9}C_2\,3^7\,a^2$; of x^3: ${}^{9}C_3\,3^6\,a^3$. Equal ($a \neq 0$) when $36 \times 3 = 84a$, so $a = 108/84 = 9/7$.

Q8. Answer (4). In $(a + b)^6 + (a - b)^6$ the odd powers of b cancel: $2[a^6 + 15a^4b^2 + 15a^2b^4 + b^6]$ with $a = \sqrt{2}$, $b = 1$: $2[8 + 60 + 30 + 1] = 198$.

Q9. Answer (1). $9^n = (1 + 8)^n = 1 + 8n + {}^{n}C_2\,8^2 + \ldots$, so $9^n - 8n - 1$ is a sum of multiples of $8^2 = 64$. In the same way $8^n = (1 + 7)^n$ gives $8^n - 7n - 1$ as a sum of multiples of $7^2 = 49$. Both statements are true.

Q10. Answer (2). The coefficient is ${}^{4}C_4 + {}^{5}C_4 + \ldots + {}^{10}C_4$. Using ${}^{n}C_4 + {}^{n}C_5 = {}^{n+1}C_5$ repeatedly (start with ${}^{4}C_4 = {}^{5}C_5$), the sum telescopes to ${}^{11}C_5 = 462$.

Q11. Answer (3). $T_4 = {}^{n}C_3\,(ax)^{n-3}\,(1/x)^3$ has power $n - 6 = 0$, so $n = 6$. Then ${}^{6}C_3\,a^3 = 20a^3 = 5/2$ gives $a^3 = 1/8$, $a = 1/2$.

Q12. Answer (4). $101^{50} - 99^{50} = (100 + 1)^{50} - (100 - 1)^{50} = 2[{}^{50}C_1\,100^{49} + {}^{50}C_3\,100^{47} + \ldots] > 2 \times 50 \times 100^{49} = 100^{50}$. So $101^{50} > 99^{50} + 100^{50}$.

Q13. Answer (2). $C_{r+1} = C_{9-r}$, so the sum is $\Sigma\,C_rC_{9-r}$, the coefficient of x^9 in $(1 + x)^{10}(1 + x)^{10} = (1 + x)^{20}$: ${}^{20}C_9 = 167960$.

Q14. Answer (3). $T_{r+1} = {}^{10}C_r\,(x/2)^{10-r}\,(-3)^r\,x^{-2r}$ has power $10 - 3r = 4$ for $r = 2$: ${}^{10}C_2 \times (1/2)^8 \times 9 = 45 \times 9/256 = 405/256$.

Q15. Answer (1). Put $x = 1$: $3^n = 6561$, so $n = 8$. The coefficient of x^r is ${}^{8}C_r\,2^r$; the ratio of consecutive coefficients $2(8 - r)/(r + 1)$ is ≥ 1 up to $r = 5$, and $r = 5$, 6 both give $56 \times 32 = 28 \times 64 = 1792$.

Q16. Answer (4). ${}^{10}C_5 = 252$ and ${}^{10}C_6 = 210$, so A is false. R is true (choosing r things is the same as leaving out $n - r$); it pairs ${}^{10}C_6$ with ${}^{10}C_4$, not with ${}^{10}C_5$.

Q17. Answer (2). x^3 comes from $1 \times {}^{10}C_3x^3$, $(-x) \times {}^{10}C_2x^2$ and $x^2 \times {}^{10}C_1x$: $120 - 45 + 10 = 85$.

Q18. Answer (1). ${}^{n}C_2 = {}^{n}C_{12}$ with $2 \neq 12$ needs $n = 14$. The middle (8th) term has coefficient ${}^{14}C_7 = 3432$.

Q19. Answer (3). $^{2n}C_n$ / $^{2n-1}C_n$ = [(2n)!/(n! n!)] × [n! (n − 1)!/(2n − 1)!] = 2n/n = 2.

Q20. Answer (2). With coefficients $^nC_{r-1}$, ^nC_r, $^nC_{r+1}$: (n − r + 1)/r = 7 and (n − r)/(r + 1) = 6. So n = 8r − 1 and n = 7r + 6, giving r = 7 and n = 55.

Q21. Answer 41. Take x^{2k} from $(1 + x^2)^5$ and x^{10-2k} from $(1 + x)^4$, which needs 10 − 2k ≤ 4, so k = 3, 4, 5: $^5C_3 \times {}^4C_4 + {}^5C_4 \times {}^4C_2 + {}^5C_5 \times {}^4C_0 = 10 + 30 + 1 = 41$.

Q22. Answer 61. $11^{20} = (1 + 10)^{20} = 1 + 2026 \times 10 +$ (terms with 10^2 and higher). So the remainder mod 100 is that of 1 + 20260 = 20261, which is 61.

Q23. Answer 1020. $(1 + 0.002)^{10} = 1 + 10(0.002) + 45(0.002)^2 + \ldots = 1 + 0.02 + 0.00018 + \ldots \approx 1.02018$ (the later terms are below 10^{-6}). So $1000 \times (1.002)^{10} \approx 1020.18$, which is 1020 to the nearest integer.

Q24. Answer 11. $^{20}C_4 = 4845$ and $^{20}C_5 = 15504$, and the coefficients rise up to r = 10 and fall symmetrically. So $^{20}C_r > 10000$ exactly for r = 5, 6, …, 15: 11 values.

Q25. Answer 171. $\Sigma\ {}^6C_k\ 2^k \times {}^7C_{5-k}\ (-1)^{5-k}$ over k = 0, …, 5: −21 + 12 × 35 − 60 × 35 + 160 × 21 − 240 × 7 + 192 × 1 = −21 + 420 − 2100 + 3360 − 1680 + 192 = 171.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: each expansion was multiplied out term by term in exact arithmetic, remainders were found with exact big integers, and sums of coefficients were added up directly.