Master Testing Agency
Practice papers, exam-standard · www.mastertestingagency.com

Complex Numbers and Quadratic Equations — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Master Testing Agency is an independent test-prep publisher, not affiliated with, endorsed by or connected to the National Testing Agency (NTA), the IITs or any other exam-conducting body.

Topics covered: Algebra of complex numbers · Modulus and argument · Argand plane and loci · Cube roots of unity · Quadratic equations · Roots and coefficients · Nature and location of roots

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. The principal argument of $z = (1 + i\sqrt{3})/(1 - i)$ is

(1) $\pi/12$

(2) $7\pi/12$

(3) $-\pi/12$

(4) $5\pi/12$

2. If $z \neq -1$ and $\mathrm{Re}((z - 1)/(z + 1)) = 0$, then z lies on

(1) the real axis

(2) the imaginary axis

(3) the circle $|z| = 1$

(4) the line $\mathrm{Re}\, z = 1$

3. The smallest positive integer n for which $((1 + i)/\sqrt{2})^n$ is a positive real number is

(1) 2

(2) 4

(3) 16

(4) 8

4. The square roots of $-5 + 12i$ are

(1) $\pm(3 + 2i)$

(2) $\pm(2 - 3i)$

(3) $\pm(2 + 3i)$

(4) $\pm(3 - 2i)$

5. If ω ($\neq 1$) is a cube root of unity, then $(1 + \omega - \omega^2)^7$ is equal to

(1) 128ω

(2) $-128\omega^2$

(3) -128ω

(4) $128\omega^2$

6. If α and β are the roots of $x^2 - px + q = 0$, the quadratic equation whose roots are α^2 and β^2 is

(1) $x^2 - (p^2 - 2q)x + q^2 = 0$

(2) $x^2 - (p^2 + 2q)x + q^2 = 0$

(3) $x^2 + (p^2 - 2q)x + q^2 = 0$

(4) $x^2 - (p^2 - 2q)x + 2q = 0$

7. Both roots of $x^2 - 2ax + a^2 + a - 7 = 0$ are real and greater than 1. Then

(1) $3 < a \leq 7$

(2) $a > 3$

(3) $a \leq 7$

(4) $1 < a < 3$

8. The roots of $z^2 - (3 + i)z + (2 + 2i) = 0$ are

(1) 1 and $2 + i$

(2) 2 and $1 + i$

(3) $1 - i$ and $2 + 2i$

(4) -2 and $-1 - i$

9. Two vertices of an equilateral triangle are 0 and $2 + 2i$, and the third vertex lies above the line joining them (anticlockwise from $2 + 2i$). The third vertex is

(1) $(1 + \sqrt{3}) + i(1 - \sqrt{3})$

(2) $2\sqrt{3} + 2i$

(3) $-(1 + \sqrt{3}) + i(1 - \sqrt{3})$

(4) $(1 - \sqrt{3}) + i(1 + \sqrt{3})$

10. If $|z_1| = 3$, $|z_2| = 4$ and $|z_1 + z_2| = 5$, then $|z_1 - z_2|$ is

(1) 1

(2) 7

(3) 5

(4) $\sqrt{7}$

11. The roots of $x^2 - (k - 3)x + k = 0$ are real and of opposite signs exactly when

(1) $k > 0$

(2) $k < 0$

(3) $k < 1$ or $k > 9$

(4) $0 < k < 9$

12. The value of $i + i^2 + i^3 + \ldots + i^{2026}$ is

(1) 0

(2) i

(3) $-1 + i$

(4) $1 - i$

13. The sum of the real roots of $3^{2x} - 10 \cdot 3^x + 9 = 0$ is

(1) 10

(2) 9

(3) 0

(4) 2

14. The number of complex numbers z satisfying $z^5 = \bar{z}$ is

(1) 5

(2) 6

(3) 7

(4) 1

15. If $|z - 3 - 4i| = 2$, the greatest value of $|z|$ is

(1) 5

(2) 7

(3) 3

(4) 9

16. If a, b, c are real and $b \neq c$, the roots of $(b - c)x^2 + (c - a)x + (a - b) = 0$ are

(1) 1 and $(a - b)/(b - c)$

(2) -1 and $(a - b)/(b - c)$

(3) 1 and $(b - c)/(a - b)$

(4) a and b

17. The set of real x satisfying $x^2 - 5|x| + 6 < 0$ is

(1) $(2, 3)$

(2) $(-3, 3)$

(3) $(-3, -2) \cup (2, 3)$

(4) $(-\infty, -3) \cup (3, \infty)$

18. Both roots of $x^2 - 2kx + 4 = 0$ are real and lie in the interval $(1, 5)$ exactly when

(1) $2 \le k < 5/2$

(2) $2 \le k < 5$

(3) $k \ge 2$

(4) $1 < k < 5/2$

19. The modulus and the principal argument of $1 + \cos(2\pi/3) + i \sin(2\pi/3)$ are

(1) 2 and $\pi/3$

(2) 1 and $\pi/3$

(3) 1 and $2\pi/3$

(4) $\sqrt{3}$ and $\pi/6$

20. If α and β are the roots of $x^2 + x + 1 = 0$, then $\alpha^{31} + \beta^{31}$ is equal to

(1) 1

(2) 2

(3) 0

(4) -1

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

---

21. If α and β are the roots of $x^2 - 5x + 3 = 0$, then $\alpha^3 + \beta^3$ is equal to _____.

Answer: _____

22. If $z = (\sqrt{3} + i)^8/(1 - i)^6$, then $|z|$ is equal to _____.

Answer: _____

23. The number of integers k for which the equation $x^2 - (k - 2)x + 4 = 0$ has no real roots is _____.

Answer: _____

24. The sum of all real roots of $|x - 2|^2 + |x - 2| - 2 = 0$ is _____.

Answer: _____

25. The number of integers a for which both roots of $x^2 - 2ax + a^2 - 1 = 0$ lie strictly between -2 and 4 is _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(2)	(3)	(4)	(3)	(2)	(1)	(1)	(2)	(4)	(3)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(2)	(3)	(4)	(3)	(2)	(1)	(3)	(1)	(2)	(4)

Q21	Q22	Q23	Q24	Q25
80	32	7	4	3

WORKED SOLUTIONS

Q1. Answer (2). $\arg(1 + i\sqrt{3}) = \pi/3$ and $\arg(1 - i) = -\pi/4$, so $\arg z = \pi/3 + \pi/4 = 7\pi/12$, which already lies in $(-\pi, \pi]$.

Q2. Answer (3). With $z = x + iy$, $(z - 1)/(z + 1) = [(x^2 + y^2 - 1) + 2iy]/[(x + 1)^2 + y^2]$. Its real part vanishes when $x^2 + y^2 = 1$: the unit circle (without the point -1).

Q3. Answer (4). $(1 + i)/\sqrt{2} = \cos(\pi/4) + i\sin(\pi/4)$, so its n-th power is $\cos(n\pi/4) + i\sin(n\pi/4)$, a positive real number when $n\pi/4$ is a multiple of 2π: $n = 8$. ($n = 4$ gives -1.)

Q4. Answer (3). $(a + ib)^2 = -5 + 12i$ gives $a^2 - b^2 = -5$ and $ab = 6$, with $a^2 + b^2 = |-5 + 12i| = 13$. So $a^2 = 4$, $b^2 = 9$, and $ab > 0$: $\pm(2 + 3i)$.

Q5. Answer (2). $1 + \omega = -\omega^2$, so $1 + \omega - \omega^2 = -2\omega^2$. Then $(-2\omega^2)^7 = -128\,\omega^{14} = -128\,\omega^2$ (as $\omega^3 = 1$, $\omega^{14} = \omega^2$).

Q6. Answer (1). $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = p^2 - 2q$ and $\alpha^2\beta^2 = q^2$, so the equation is $x^2 - (p^2 - 2q)x + q^2 = 0$.

Q7. Answer (1). The roots are $a \pm \sqrt{(7 - a)}$, real when $a \le 7$. The smaller root exceeds 1 when $\sqrt{(7 - a)} < a - 1$, which needs $a > 1$ and $7 - a < (a - 1)^2$, i.e. $a^2 - a - 6 = (a - 3)(a + 2) > 0$, so $a > 3$. Together $3 < a \le 7$.

Q8. Answer (2). Sum $3 + i$ and product $2 + 2i$: $2 + (1 + i) = 3 + i$ and $2(1 + i) = 2 + 2i$. (Or the discriminant $(3 + i)^2 - 4(2 + 2i) = -2i = (1 - i)^2$, giving $z = [(3 + i) \pm (1 - i)]/2$.)

Q9. Answer (4). Rotate $2 + 2i$ about 0 through $\pi/3$: $(2 + 2i)(\tfrac{1}{2} + i\sqrt{3}/2) = (1 - \sqrt{3}) + i(1 + \sqrt{3})$.

Q10. Answer (3). $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2) = 50$, so $|z_1 - z_2|^2 = 50 - 25 = 25$ and $|z_1 - z_2| = 5$. (Here z_1 and z_2 are at right angles.)

Q11. Answer (2). Roots of opposite signs need a negative product: $k < 0$. Then the discriminant $(k - 3)^2 - 4k$ is automatically positive.

Q12. Answer (3). Every four consecutive powers of i add to 0. $2026 = 4 \times 506 + 2$, so the sum equals the first two terms: $i + i^2 = -1 + i$.

Q13. Answer (4). Put $t = 3^x > 0$: $t^2 - 10t + 9 = 0$ gives $t = 1$ or 9, so $x = 0$ or 2. Sum 2.

Q14. Answer (3). Taking moduli, $|z|^5 = |z|$, so $|z| = 0$ or 1. $z = 0$ works. If $|z| = 1$ then $\bar{z} = 1/z$, so $z^6 = 1$: six roots of unity. Total 7.

Q15. Answer (2). z lies on the circle of radius 2 with centre $3 + 4i$, which is 5 from the origin. The farthest point of the circle is $5 + 2 = 7$ from the origin.

Q16. Answer (1). The coefficients add to zero, so $x = 1$ is a root; the product of the roots is $(a - b)/(b - c)$, which is then the other root.

Q17. Answer (3). With $t = |x|$: $t^2 - 5t + 6 = (t - 2)(t - 3) < 0$, so $2 < |x| < 3$, i.e. $x \in (-3, -2) \cup (2, 3)$.

Q18. Answer (1). With $f(x) = x^2 - 2kx + 4$: real roots need $k^2 - 4 \geq 0$ ($|k| \geq 2$); the vertex k must lie in $(1, 5)$; and $f(1) = 5 - 2k > 0$, $f(5) = 29 - 10k > 0$, so $k < 5/2$. Together $2 \leq k < 5/2$ ($k = 2$ gives the double root 2).

Q19. Answer (2). $1 + \cos\theta + i\sin\theta = 2\cos(\theta/2)[\cos(\theta/2) + i\sin(\theta/2)]$. With $\theta = 2\pi/3$: modulus $2\cos(\pi/3) = 1$ and argument $\pi/3$. (Directly: $\frac{1}{2} + i\sqrt{3}/2$.)

Q20. Answer (4). The roots are ω and ω^2. $\alpha^{31} + \beta^{31} = \omega^{31} + \omega^{62} = \omega + \omega^2 = -1$ (as $31 = 3 \times 10 + 1$ and $62 = 3 \times 20 + 2$).

Q21. Answer 80. $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 125 - 3 \times 3 \times 5 = 125 - 45 = 80$.

Q22. Answer 32. $|\sqrt{3} + i| = 2$ and $|1 - i| = \sqrt{2}$, so $|z| = 2^8/(\sqrt{2})^6 = 256/8 = 32$.

Q23. Answer 7. No real roots: $(k - 2)^2 - 16 < 0$, i.e. $|k - 2| < 4$, $-2 < k < 6$. Integers $-1, 0, 1, 2, 3, 4, 5$: 7.

Q24. Answer 4. With $t = |x - 2| \geq 0$: $t^2 + t - 2 = (t + 2)(t - 1) = 0$, so $t = 1$ ($t = -2$ is impossible). $x - 2 = \pm 1$ gives $x = 1, 3$. Sum 4.

Q25. Answer 3. The roots are $a - 1$ and $a + 1$. Need $-2 < a - 1$ and $a + 1 < 4$: $-1 < a < 3$. Integers 0, 1, 2: 3.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: complex expressions were evaluated numerically against each option, loci tested on sample points, solutions counted by a numerical root search, and parameter conditions tested by solving the equation for a fine grid of parameter values.