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# Differential Equations — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

| DURATION | MAXIMUM MARKS | QUESTIONS | NAME / ROLL NO. |
|---|---|---|---|
| 60 minutes | 100 | 25 (20 + 5) | |

## GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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**Topics covered:** Order and degree · Variables separable · Homogeneous equations · Linear equations

## SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

**1.** The order and the degree of the differential equation $[1 + (dy/dx)^2]^{3/2} = 5\, d^2y/dx^2$ are respectively

(1) 2 and 2

(2) 2 and 3/2

(3) 3 and 2

(4) 2 and 1

**2.** The solution of $dy/dx = (1 + y^2)/(1 + x^2)$ with $y(0) = 1$ is

(1) $y = (1 - x)/(1 + x)$

(2) $y = (1 + x)/(1 - x)$

(3) $y = 1 + x$

(4) $y = (1 + x)/(1 + x^2)$

**3.** The solution of $dy/dx = y \log_e y$ $(y > 1)$ with $y(0) = e$ is

(1) $y = e^x$

(2) $y = e^{(x + 1)}$

(3) $y = e^{(e^x)}$

(4) $y = e^{(e^{-x})}$

**4.** The solution of $x\, dy/dx = y + \sqrt{(x^2 + y^2)}$, $x > 0$, with $y(1) = 0$ is

(1) $y = x^2 - 1$

(2) $y = (x - 1)^2/2$

(3) $y = (1 - x^2)/2$

(4) $y = (x^2 - 1)/2$

**5.** The solution of $(x - y)\, dy = (x + y)\, dx$ with $y(1) = 0$ is

(1) $\tan^{-1}(y/x) = \ln(x^2 + y^2)$

(2) $\tan^{-1}(y/x) = (1/2) \ln(x^2 + y^2)$

(3) $\tan^{-1}(x/y) = (1/2) \ln(x^2 + y^2)$

(4) $\tan^{-1}(y/x) = -(1/2) \ln(x^2 + y^2)$

**6.** The solution of $dy/dx + 2y = e^{-x}$ with $y(0) = 2$ is

(1) $y = e^{-x} + e^{-2x}$

(2) $y = 2e^{-2x}$

(3) $y = 2e^{-x}$

(4) $y = e^{-x}(2 - x)$

**7.** The solution of $dy/dx - y/x = xe^x$, $x > 0$, with $y(1) = e$ is

(1) $y = e^x$

(2) $y = x^2e^x$

(3) $y = xe^x$

(4) $y = (x + 1)e^x - e$

**8.** If $(1 + x^2)\,dy/dx + 2xy = \cos x$ and $y(0) = 0$, then $y(\pi/2)$ is equal to

(1) $4/\pi^2$

(2) $4/(4 + \pi^2)$

(3) $1/(1 + \pi^2)$

(4) $2/(4 + \pi^2)$

**9.** Statement I: The degree of the differential equation $(d^2y/dx^2)^2 + (dy/dx)^3 = x$ is 3.
Statement II: The differential equation $dy/dx = (x + y)/(x - y)$ is homogeneous.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

**10.** A colony of bacteria grows at a rate proportional to the number present. It doubles in 3 hours. The time (in hours) in which it becomes 8 times the initial number is

(1) 6

(2) 12

(3) 24

(4) 9

**11.** The solution curve of $dy/dx = 2x(y - 1)$ that passes through the point (0, 3) is

(1) $y = 1 + 2e^{(x^2)}$

(2) $y = 3e^{(x^2)}$

(3) $y = 2 + e^{(x^2)}$

(4) $y = 1 + 2e^{(2x)}$

**12.** The solution curve of $(x + y^2)\,dy = y\,dx$, $y > 0$, that passes through the point (2, 1) is

(1) $x = y^2 - y$

(2) $x = 2y^2$

(3) $x = y^2 + y$

(4) $y = x^2 + x$

**13.** A curve passes through (1, 1), and at each point (x, y) on it ($x > 0$) the slope of the tangent is $y/x + 1$. Then $y(e)$ is equal to

(1) e

(2) $e^2$

(3) e + 1

(4) 2e

**14.** The solution of $dy/dx = \cos(x + y)$ with $y(0) = 0$ is

(1) $\tan((x + y)/2) = x$

(2) $\tan(x + y) = x$

(3) $\sin((x + y)/2) = x$

(4) $\tan((x + y)/2) = 2x$

**15.** If $y = y(x)$ is the solution of $dy/dx + y = x$ with $y(0) = 0$, then $y(1)$ is equal to

(1) $1 - 1/e$

(2) $1/e$

(3) $e - 2$

(4) $2/e$

**16.** Assertion A: The solution curve of $dy/dx = y/x$ through the point $(1, 2)$ also passes through the point $(3, 6)$.
Reason R: Every solution curve of $dy/dx = y/x$ $(x \neq 0)$ is part of a straight line through the origin.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

**17.** The solution of $x\,dy/dx = y(\log_e y - \log_e x + 1)$, $x, y > 0$, with $y(1) = e^2$ is

(1) $y = x^2e^x$

(2) y = e^(2x)

(3) y = xe^(x + 1)

(4) y = xe^(2x)

**18.** The solution of $dy/dx = y + 2x$ with $y(0) = -2$ is

(1) $y = -2e^x$

(2) $y = -2(x + 1)$

(3) $y = 2x - 2$

(4) $y = -2(x + 1)e^x$

**19.** The solution of $dy/dx = (2x + 1)/(2y)$, $y > 0$, with $y(0) = 2$ has $y(3)$ equal to

(1) $2\sqrt{3}$

(2) 3

(3) 4

(4) 5

**20.** If $dy/dx + y/x = \sin x$ $(x > 0)$ and $y(\pi) = 1$, then $y(\pi/2)$ is equal to

(1) $\pi/2$

(2) 1

(3) 0

(4) $2/\pi$

## SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

**21.** If m and n are the order and the degree of the differential equation $(d^3y/dx^3)^{(2/3)} = 1 + dy/dx$, then m + n is equal to _____.

Answer: _____

**22.** If $y = y(x)$ satisfies $dy/dx = 2y/(x + 1)$, $x > -1$, and $y(0) = 1$, then $y(3)$ is equal to _____.

Answer: _____

**23.** If $y = y(x)$ satisfies $dy/dx - 3y = 6$ and $y(0) = 0$, then $y(\log_e 2)$ is equal to _____.

Answer: _____

**24.** A population $p(t)$ satisfies $dp/dt = 0.5p - 300$ with $p(0) = 500$. The population becomes zero at time $t = 2 \log_e k$. Then $k$ is equal to _____.

Answer: _____

**25.** Let $y = y(x)$, $x > 0$, be the solution of $dy/dx = (x^2 + y^2)/(xy)$ with $y(1) = 1$. Then $(y(e))^2/e^2$ is equal to _____.

Answer: _____

**END OF THE QUESTION PAPER**

## ANSWER KEY

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (2) | (1) | (3) | (2) | (3) | (4) |

| Q11 | Q12 | Q13 | Q14 | Q15 | Q16 | Q17 | Q18 | Q19 | Q20 |
|---|---|---|---|---|---|---|---|---|---|
| (1) | (3) | (4) | (1) | (2) | (1) | (4) | (2) | (3) | (4) |

| Q21 | Q22 | Q23 | Q24 | Q25 |
|---|---|---|---|---|
| 5 | 16 | 14 | 6 | 3 |

## WORKED SOLUTIONS

**Q1. Answer (1).** Squaring removes the fractional power: $[1 + (dy/dx)^2]^3 = 25\,(d^2y/dx^2)^2$. The highest derivative is $d^2y/dx^2$ (order 2) and it appears to the power 2 (degree 2).

**Q2. Answer (2).** $dy/(1 + y^2) = dx/(1 + x^2)$ gives $\tan^{-1}y = \tan^{-1}x + C$, with $C = \pi/4$ from $y(0) = 1$. So $y = \tan(\tan^{-1}x + \pi/4) = (x + 1)/(1 - x)$.

**Q3. Answer (3).** $dy/(y \ln y) = dx$ gives $\ln(\ln y) = x + C$. At $x = 0$, $\ln(\ln e) = 0$, so $C = 0$: $\ln y = e^x$, $y = e\text{^}(e^x)$.

**Q4. Answer (4).** Put $y = vx$: $x\,dv/dx = \sqrt{(1 + v^2)}$, so $\ln(v + \sqrt{(1 + v^2)}) = \ln x + C$, with $C = 0$ from $v(1) = 0$. Then $y + \sqrt{(x^2 + y^2)} = x^2$, i.e. $x^2 + y^2 = (x^2 - y)^2$, which simplifies to $y = (x^2 - 1)/2$.

**Q5. Answer (2).** $dy/dx = (x + y)/(x - y)$. With $y = vx$: $v + x\,dv/dx = (1 + v)/(1 - v)$, so $(1 - v)\,dv/(1 + v^2) = dx/x$. Integrating: $\tan^{-1}v - (1/2)\ln(1 + v^2) = \ln x + C$, i.e. $\tan^{-1}(y/x) = (1/2)\ln(x^2 + y^2) + C$, and $C = 0$ from $y(1) = 0$.

**Q6. Answer (1).** Integrating factor $e^{2x}$: $(ye^{2x})' = e^x$, so $ye^{2x} = e^x + C$ and $y = e^{-x} + Ce^{-2x}$. $y(0) = 1 + C = 2$ gives $C = 1$.

**Q7. Answer (3).** Integrating factor $1/x$: $(y/x)' = e^x$, so $y/x = e^x + C$. $y(1) = e$ gives $C = 0$, so $y = xe^x$.

**Q8. Answer (2).** The left side is $d/dx[(1 + x^2)y]$, so $(1 + x^2)y = \sin x + C$, and $C = 0$ from $y(0) = 0$. At $x = \pi/2$: $y = 1/(1 + \pi^2/4) = 4/(4 + \pi^2)$.

**Q9. Answer (3).** Statement I is false: the degree is the power of the highest derivative $d^2y/dx^2$, which is 2. Statement II is true: $(x + y)/(x - y)$ is unchanged when $x$, $y$ are replaced by $\lambda x$, $\lambda y$.

**Q10. Answer (4).** $dN/dt = kN$ gives $N = N_0e^{kt}$ with $e^{3k} = 2$. $N = 8N_0$ needs $e^{kt} = 8 = 2^3 = e^{9k}$, so $t = 9$ hours.

**Q11. Answer (1).** $dy/(y - 1) = 2x\,dx$ gives $\ln|y - 1| = x^2 + C$, so $y - 1 = Ae\text{^}(x^2)$; at $(0, 3)$, $A = 2$. So $y = 1 + 2e\text{^}(x^2)$.

**Q12. Answer (3).** Treat $x$ as a function of $y$: $dx/dy - x/y = y$, a linear equation with integrating factor $1/y$. $(x/y)' = 1$, so $x/y = y + C$; at $(2, 1)$, $C = 1$. So $x = y^2 + y$.

**Q13. Answer (4).** $dy/dx - y/x = 1$ has integrating factor $1/x$: $(y/x)' = 1/x$, so $y/x = \ln x + C$, with $C = 1$ from $(1, 1)$. So $y = x \ln x + x$ and $y(e) = e + e = 2e$.

**Q14. Answer (1).** Put $v = x + y$: $dv/dx = 1 + \cos v = 2\cos^2(v/2)$, so $(1/2)\sec^2(v/2)\,dv = dx$ and $\tan(v/2) = x + C$. At $x = 0$, $v = 0$, so $C = 0$: $\tan((x + y)/2) = x$.

**Q15. Answer (2).** Integrating factor $e^x$: $(ye^x)' = xe^x$, so $ye^x = (x - 1)e^x + C$ and $y = x - 1 + Ce^{-x}$, with $C = 1$ from $y(0) = 0$. So $y(1) = 0 + 1/e = 1/e$.

**Q16. Answer (1).** $dy/y = dx/x$ gives $y = Cx$, a line through the origin (R is true). Through $(1, 2)$, $C = 2$: $y = 2x$, which passes through $(3, 6)$. So A is true and follows from R.

**Q17. Answer (4).** With $y = vx$: $x\,dv/dx + v = v(\ln v + 1)$, so $x\,dv/dx = v \ln v$ and $dv/(v \ln v) = dx/x$. Then $\ln(\ln v) = \ln x + C$, i.e. $\ln v = kx$. From $v(1) = e^2$, $k = 2$: $y = xe^{\wedge}(2x)$.

**Q18. Answer (2).** $dy/dx - y = 2x$ with integrating factor $e^{-x}$: $ye^{-x} = \int 2xe^{-x}\,dx = -2(x + 1)e^{-x} + C$, so $y = -2(x + 1) + Ce^{x}$. $y(0) = -2 + C = -2$ gives $C = 0$.

**Q19. Answer (3).** $2y\,dy = (2x + 1)\,dx$ gives $y^2 = x^2 + x + C$, with $C = 4$ from $y(0) = 2$. At $x = 3$: $y^2 = 9 + 3 + 4 = 16$, $y = 4$.

**Q20. Answer (4).** Integrating factor $x$: $(xy)' = x \sin x$, so $xy = \sin x - x \cos x + C$. At $x = \pi$: $\pi = 0 + \pi + C$, $C = 0$. At $x = \pi/2$: $(\pi/2)y = 1$, $y = 2/\pi$.

**Q21. Answer 5.** Cubing: $(d^3y/dx^3)^2 = (1 + dy/dx)^3$. The order is 3 (third derivative) and the degree is 2 (its power). $m + n = 5$.

**Q22. Answer 16.** $dy/y = 2\,dx/(x + 1)$ gives $\ln y = 2 \ln(x + 1) + C$, with $C = 0$: $y = (x + 1)^2$, so $y(3) = 16$.

**Q23. Answer 14.** $(ye^{-3x})' = 6e^{-3x}$, so $ye^{-3x} = -2e^{-3x} + C$ and $y = -2 + Ce^{3x}$, with $C = 2$. Then $y(\ln 2) = -2 + 2 \times 8 = 14$.

**Q24. Answer 6.** $dp/(p - 600) = 0.5\,dt$ gives $p - 600 = -100e^{\wedge}(t/2)$. $p = 0$ when $e^{\wedge}(t/2) = 6$, i.e. $t = 2 \ln 6$. So $k = 6$.

**Q25. Answer 3.** With $y = vx$: $v + x\,dv/dx = 1/v + v$, so $v\,dv = dx/x$ and $v^2/2 = \ln x + 1/2$. So $y^2 = x^2(2 \ln x + 1)$, and at $x = e$, $y^2/e^2 = 3$.

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