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Integral Calculus — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Standard integrals and substitution · Integration by parts · Partial fractions · Definite integrals and properties · Area under curves

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. $\int dx/(x^2 + 6x + 13)$ is equal to

(1) $\tan^{-1}((x + 3)/2) + C$

(2) $(1/4)\tan^{-1}((x + 3)/2) + C$

(3) $(1/2)\tan^{-1}((x + 3)/2) + C$

(4) $(1/2)\tan^{-1}((x - 3)/2) + C$

2. $\int x^3 e^{\wedge}(x^2)\,dx$ is equal to

(1) $(1/2)\,e^{\wedge}(x^2)(x^2 - 1) + C$

(2) $e^{\wedge}(x^2)(x^2 - 1) + C$

(3) $(1/2)\,e^{\wedge}(x^2)(x^2 + 1) + C$

(4) $(1/2)\,x^2\,e^{\wedge}(x^2) + C$

3. $\int dx/[(x - 1)(x - 2)(x - 3)]$ is equal to

(1) $\ln|x - 1| - 2\ln|x - 2| + \ln|x - 3| + C$

(2) $(1/2)\ln|x - 1| + \ln|x - 2| + (1/2)\ln|x - 3| + C$

(3) $-(1/2)\ln|x - 1| + \ln|x - 2| - (1/2)\ln|x - 3| + C$

(4) $(1/2)\ln|x - 1| - \ln|x - 2| + (1/2)\ln|x - 3| + C$

4. $\int dx/(1 + e^x)$ is equal to

(1) $x + \ln(1 + e^x) + C$

(2) $x - \ln(1 + e^x) + C$

(3) $\ln(1 + e^x) + C$

(4) $\ln(1 + e^{-x}) + C$

5. $\int \sin^{-1}x\,dx$, $|x| < 1$, is equal to

(1) $x\sin^{-1}x - \sqrt{(1 - x^2)} + C$

(2) $\sin^{-1}x + x\sqrt{(1 - x^2)} + C$

(3) $x\sin^{-1}x + \sqrt{(1 - x^2)} + C$

(4) $x\sin^{-1}x + 1/\sqrt{(1 - x^2)} + C$

6. The value of $\int_0^{\wedge}\pi\, x\sin^3x\,dx$ is

(1) $2\pi/3$

(2) $\pi/3$

(3) $4\pi/3$

(4) $\pi^2/3$

7. The value of $\int$ from $-\pi/4$ to $\pi/4$ of $(x^3 + x\cos x + \tan^5x + 1)\,dx$ is

(1) $\pi/4$

(2) 1

(3) 0

(4) $\pi/2$

8. The value of $\int_0^2 |x^2 - 1|\, dx$ is

(1) 4/3

(2) 2

(3) 8/3

(4) 3

9. For $x > 1$, the derivative with respect to x of $\int$ from x^2 to x^3 of $dt/\ln t$ is

(1) $(x^3 - x^2)/\ln x$

(2) $(x - 1)/\ln x$

(3) $(x^2 - x)/\ln x$

(4) $(x^2 + x)/\ln x$

10. The value of $\int_0^1 x(1 - x)^9\, dx$ is

(1) 1/90

(2) 1/100

(3) 1/11

(4) 1/110

11. For $x > 0$, $\int dx/[x(x^4 + 1)]$ is equal to

(1) $(1/4)\ln(x^4/(x^4 + 1)) + C$

(2) $\ln(x^4/(x^4 + 1)) + C$

(3) $(1/4)\ln((x^4 + 1)/x^4) + C$

(4) $(1/2)\ln(x^2/(x^4 + 1)) + C$

12. The area of the region enclosed by the parabola $y = x^2$ and the line $y = x + 2$ is

(1) 9

(2) 9/2

(3) 7/2

(4) 5

13. The area of the region bounded by $y = \ln x$, the x-axis and the line $x = e$ is

(1) $e - 1$

(2) e

(3) $1/e$

(4) 1

14. The area of the region bounded by the parabola $y^2 = 4x$ and the line $y = 2x - 4$ is

(1) 9

(2) 6

(3) 18

(4) 27/2

15. Statement I: $1 < \int_0^1 e^{(x^2)}\,dx < e$.
Statement II: $\int_0^{(\pi/2)} \sqrt{(\sin x)}\,dx > \pi/2$.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

16. Assertion A: $\int_0^{(\pi/2)} \ln(\tan x)\,dx = 0$.
Reason R: $\int_0^a f(x)\,dx = \int_0^a f(a - x)\,dx$.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

17. For $0 < x < \pi/2$, $\int dx/(\sin x + \cos x)$ is equal to

(1) $\sqrt{2}\,\ln|\tan(x/2 + \pi/8)| + C$

(2) $(1/\sqrt{2})\,\ln|\tan(x/2 - \pi/8)| + C$

(3) $(1/\sqrt{2})\,\ln|\tan(x/2 + \pi/8)| + C$

(4) $(1/\sqrt{2})\,\ln|\tan(x + \pi/4)| + C$

18. The value of $\int_0^1 dx/(e^x + e^{-x})$ is

(1) $\tan^{-1}e$

(2) $\pi/4$

(3) $\tan^{-1}e + \pi/4$

(4) $\tan^{-1}e - \pi/4$

19. The value of $\int_0^1 \tan^{-1}x\,dx$ is

(1) $\pi/4 + (1/2)\ln 2$

(2) $\pi/4 - (1/2)\ln 2$

(3) $\pi/4 - \ln 2$

(4) $\pi/2 - \ln 2$

20. Let f be a continuous function with $\int_0^{(x^2)} f(t)\,dt = x^3$ for every $x > 0$. Then $f(4)$ is equal to

(1) 3/2

(2) 6

(3) 3

(4) 3/4

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

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21. The value of $\int_0^4 (|x - 1| + |x - 3|)\,dx$ is _____.

Answer: _____

22. The value of $3\int_{-1}^{2} x|x|\,dx$ is _____.

Answer: _____

23. If A is the area of the region bounded by the curve $y = \sqrt{x}$ and the line $y = x/2$, then 6A is equal to _____.

Answer: _____

24. If $\int_{1}^{e} (\ln x)^2\,dx = ae + b$ for integers a and b, then $a^2 + b^2$ is equal to _____.

Answer: _____

25. The value of $\int_{0}^{2} x^3\,dx + \int_{0}^{8} \sqrt[3]{x}\,dx$ is _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(3)	(1)	(4)	(2)	(3)	(1)	(4)	(2)	(3)	(4)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(1)	(2)	(4)	(1)	(2)	(1)	(3)	(4)	(2)	(3)

Q21	Q22	Q23	Q24	Q25
10	7	8	5	16

WORKED SOLUTIONS

Q1. Answer (3). $x^2 + 6x + 13 = (x + 3)^2 + 2^2$, and $\int dx/(u^2 + a^2) = (1/a)\tan^{-1}(u/a)$: $(1/2)\tan^{-1}((x + 3)/2) + C$.

Q2. Answer (1). Put $t = x^2$, $dt = 2x\,dx$: $(1/2)\int t\,e^t\,dt = (1/2)(t - 1)e^t + C = (1/2)\,e^{\wedge}(x^2)(x^2 - 1) + C$.

Q3. Answer (4). $1/[(x - 1)(x - 2)(x - 3)] = A/(x - 1) + B/(x - 2) + C/(x - 3)$ with $A = 1/[(-1)(-2)] = 1/2$, $B = 1/[(1)(-1)] = -1$, $C = 1/[(2)(1)] = 1/2$.

Q4. Answer (2). $1/(1 + e^x) = 1 - e^x/(1 + e^x)$, and $\int e^x/(1 + e^x)\,dx = \ln(1 + e^x)$. So the integral is $x - \ln(1 + e^x) + C$.

Q5. Answer (3). By parts with $u = \sin^{-1}x$, $dv = dx$: $x\sin^{-1}x - \int x/\sqrt{(1 - x^2)}\,dx = x\sin^{-1}x + \sqrt{(1 - x^2)} + C$.

Q6. Answer (1). With $x \to \pi - x$ ($\sin(\pi - x) = \sin x$): $I = \int_0{\wedge}\pi\,(\pi - x)\sin^3x\,dx$, so $2I = \pi\int_0{\wedge}\pi \sin^3x\,dx = \pi \times 4/3$ and $I = 2\pi/3$. ($\int_0{\wedge}\pi \sin^3x\,dx = \int_{-1}^{1}(1 - u^2)\,du = 4/3$ with $u = \cos x$.)

Q7. Answer (4). x^3, $x\cos x$ and $\tan^5x$ are odd functions, so they integrate to 0 over the symmetric interval. What is left is $\int 1\,dx = \pi/2$.

Q8. Answer (2). $\int_0^1 (1 - x^2)\,dx + \int_1^2 (x^2 - 1)\,dx = 2/3 + [x^3/3 - x]_1^2 = 2/3 + (2/3 + 2/3) = 2$.

Q9. Answer (3). By the fundamental theorem of calculus: $3x^2/\ln(x^3) - 2x/\ln(x^2) = 3x^2/(3\ln x) - 2x/(2\ln x) = (x^2 - x)/\ln x$.

Q10. Answer (4). Replace x by $1 - x$: $\int_0^1 (1 - x)x^9\,dx = \int_0^1 (x^9 - x^{10})\,dx = 1/10 - 1/11 = 1/110$.

Q11. Answer (1). Write it as $\int x^3\,dx/[x^4(x^4 + 1)]$ and put $t = x^4$: $(1/4)\int dt/[t(t + 1)] = (1/4)\int [1/t - 1/(t + 1)]\,dt = (1/4)\ln(x^4/(x^4 + 1)) + C$.

Q12. Answer (2). They meet where $x^2 = x + 2$: $x = -1$ and 2. Area $= \int_{-1}^{2}(x + 2 - x^2)\,dx = [x^2/2 + 2x - x^3/3]_{-1}^{2} = 10/3 + 7/6 = 9/2$.

Q13. Answer (4). $\ln x \geq 0$ for $1 \leq x \leq e$, so the area is $\int_1^e \ln x\,dx = [x\ln x - x]_1^e = (e - e) - (0 - 1) = 1$.

Q14. Answer (1). Substitute $x = (y + 4)/2$: $y^2 = 2y + 8$, so $y = -2$ and 4. Integrating along y: $\int_{-2}^{4}[(y + 4)/2 - y^2/4]\,dy = [y^2/4 + 2y - y^3/12]_{-2}^{4} = 20/3 + 7/3 = 9$.

Q15. Answer (2). Statement I: $1 < e^{\wedge}(x^2) < e$ for $0 < x < 1$, so the integral lies between 1 and e: true. Statement II: $\sqrt{(\sin x)} \leq 1$, so the integral is at most $\pi/2$ (and in fact less): false.

Q16. Answer (1). By R with $a = \pi/2$, $I = \int_0{\wedge}(\pi/2)\ln(\cot x)\,dx = -I$, so $I = 0$. Both are true and R is the correct explanation of A.

Q17. Answer (3). $\sin x + \cos x = \sqrt{2}\sin(x + \pi/4)$, and $\int \operatorname{cosec} u\,du = \ln|\tan(u/2)|$. So the integral is $(1/\sqrt{2})\ln|\tan(x/2 + \pi/8)| + C$.

Q18. Answer (4). Multiply above and below by e^x and put $t = e^x$: $\int_1^e dt/(t^2 + 1) = \tan^{-1}e - \tan^{-1}1 = \tan^{-1}e - \pi/4$.

Q19. Answer (2). By parts: $[x \tan^{-1}x]_0^1 - \int_0^1 x/(1 + x^2)\,dx = \pi/4 - (1/2)\ln 2$.

Q20. Answer (3). Differentiate both sides: $f(x^2) \times 2x = 3x^2$, so $f(x^2) = 3x/2$. With $x = 2$: $f(4) = 3$.

Q21. Answer 10. $\int_0^4 |x - 1|\,dx = 1/2 + 9/2 = 5$ and $\int_0^4 |x - 3|\,dx = 9/2 + 1/2 = 5$ (two triangles each). Total 10.

Q22. Answer 7. $\int_{-1}^{0} (-x^2)\,dx + \int_0^2 x^2\,dx = -1/3 + 8/3 = 7/3$. Three times this is 7.

Q23. Answer 8. $\sqrt{x} = x/2$ at $x = 0$ and $x = 4$, with $\sqrt{x}$ above. $A = \int_0^4 (\sqrt{x} - x/2)\,dx = 16/3 - 4 = 4/3$, so $6A = 8$.

Q24. Answer 5. By parts twice, $\int (\ln x)^2\,dx = x(\ln x)^2 - 2x \ln x + 2x$. From 1 to e: $(e - 2e + 2e) - 2 = e - 2$. So $a = 1$, $b = -2$ and $a^2 + b^2 = 5$.

Q25. Answer 16. $y = x^3$ and $x = \sqrt[3]{y}$ are the same curve. The two integrals are the areas below the curve (from $x = 0$ to 2) and to its left (from $y = 0$ to 8), which fill the rectangle 2×8. So the sum is 16. (Directly: $4 + 12 = 16$.)

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: each option of an indefinite integral was differentiated numerically and compared with the integrand, and definite integrals and areas were evaluated by Simpson's rule.