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Limit, Continuity and Differentiability — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Limits · Continuity · Differentiability and derivatives · Rate of change and monotonicity · Maxima and minima

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. The value of $\lim_{x \to 0} \frac{e^{3x} - 1 - 3x}{x^2}$ is

(1) 9/2

(2) 3/2

(3) 9

(4) 3

2. The value of $\lim_{x \to 0} \frac{e^{x} - e^{-x} - 2x}{x - \sin x}$ is

(1) 1

(2) 1/3

(3) 2

(4) 3

3. The value of $\lim_{x \to \infty} (1 + 2/x)^{3x}$ is

(1) e^3

(2) e^6

(3) e^(2/3)

(4) e^2

4. The value of $\lim_{x \to 2} \frac{x^5 - 32}{x^3 - 8}$ is

(1) 4

(2) 10/3

(3) 32/3

(4) 20/3

5. The value of $\lim_{x \to 0} \frac{\sin 3x - 3 \sin x}{x^3}$ is

(1) 4

(2) −4

(3) −3

(4) −9/2

6. The value of $\lim_{x \to \pi/2} \frac{1 - \sin x}{(\pi/2 - x)^2}$ is

(1) 1/2

(2) 1

(3) 0

(4) 1/4

7. Let $f(x) = \frac{\sqrt{(1 + kx)} - \sqrt{(1 - kx)}}{x}$ for $x \neq 0$ (near 0) and $f(0) = 2k + 1$. If f is continuous at x = 0, then k is equal to

(1) 1

(2) 0

(3) −1

(4) −1/2

8. Let $f(x) = (1 - \cos ax)/x^2$ for $x \neq 0$ and $f(0) = 8$, where $a > 0$. If f is continuous at $x = 0$, then a is equal to

(1) 2

(2) 8

(3) 16

(4) 4

9. If $y = \log_e(x + \sqrt{(x^2 + 1)})$, then $(x^2 + 1)\, d^2y/dx^2 + x\, dy/dx$ is equal to

(1) 1

(2) 0

(3) y

(4) −1

10. For $0 < x < \pi/4$, the derivative of $\tan^{-1}[(\cos x - \sin x)/(\cos x + \sin x)]$ with respect to x is

(1) −1

(2) 1

(3) 1/2

(4) 0

11. A ladder 5 m long leans against a vertical wall. Its foot slides away from the wall along the floor at 2 m/s. When the foot is 4 m from the wall, the top of the ladder slides down the wall at

(1) 3/2 m/s

(2) 2 m/s

(3) 8/3 m/s

(4) 4/3 m/s

12. The function $f(x) = x^3 - 3x^2 - 9x + 5$ is strictly decreasing on the interval

(1) (−3, 1)

(2) (−∞, −1)

(3) (3, ∞)

(4) (−1, 3)

13. The greatest value of $f(x) = x^2e^{-x}$ for $x \geq 0$ is

(1) 1/e

(2) $4/e^2$

(3) 2/e

(4) 4/e

14. A right circular cylinder of greatest volume is inscribed in a sphere of radius √3. The height of the cylinder is

(1) 2

(2) √3

(3) 3

(4) 2√3

15. Statement I: $\lim_{x \to 0} x \sin(1/x) = 0$.
Statement II: $\lim_{x \to 0} \sin(1/x)$ exists.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

16. Assertion A: The function $f(x) = |x|$ has a local minimum at $x = 0$.
Reason R: $f'(0) = 0$.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

17. The value of $\lim_{x \to 0} (2^x - 1)/(3^x - 1)$ is

(1) $\log_2 3$

(2) 2/3

(3) 1

(4) $\log_3 2$

18. If $x = a(t - \sin t)$ and $y = a(1 - \cos t)$, $a > 0$, then d^2y/dx^2 at $t = \pi/2$ is

(1) $-1/a$

(2) $1/a$

(3) $-1/(2a)$

(4) $-2/a$

19. The absolute maximum value of $f(x) = 2x^3 - 15x^2 + 36x + 1$ on the interval $[1, 5]$ is

(1) 29

(2) 24

(3) 56

(4) 28

20. Let $f(x) = \sin(\log_e x)$, $x > 0$, and $y = f((2x + 3)/(3 - 2x))$. Then dy/dx at $x = 0$ is

(1) 3/4

(2) 1

(3) 2/3

(4) 4/3

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

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21. If $L = \lim_{x \to 0} (\tan x - \sin x)/x^3$, then $12L$ is equal to _____.

Answer: _____

22. Let $f(x) = x^4 - 8x^2 + 5$. The sum of the absolute values of all the local maximum and local minimum values of f is _____.

Answer: _____

23. A particle moves along the curve $y = x^3$, and its x-coordinate increases at 2 units per second. The rate (in units per second) at which its y-coordinate increases when $x = 3$ is _____.

Answer: _____

24. Let $f(x) = ax + 1$ for $x \le 1$, $f(x) = x^2 + b$ for $1 < x \le 3$, and $f(x) = 5x - a$ for $x > 3$. If f is continuous on $\mathbb{R}$, then $a^2 + b^2$ is equal to _____.

Answer: _____

25. The function $f(x) = x^3 + ax^2 + bx + 5$ has local extrema at $x = 1$ and $x = 5$. The local maximum value of f is _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(1)	(3)	(2)	(4)	(2)	(1)	(3)	(4)	(2)	(1)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(3)	(4)	(2)	(1)	(2)	(3)	(4)	(1)	(3)	(4)

Q21	Q22	Q23	Q24	Q25
6	27	54	18	12

WORKED SOLUTIONS

Q1. Answer (1). $e^{3x} = 1 + 3x + 9x^2/2 + O(x^3)$, so the numerator is $9x^2/2 + O(x^3)$ and the limit is 9/2. (Or L'Hospital twice: $9e^{3x}/2 \to 9/2$.)

Q2. Answer (3). $e^x - e^{-x} = 2(x + x^3/6 + \ldots)$, so the numerator is $x^3/3 + O(x^5)$; $x - \sin x = x^3/6 + O(x^5)$. The limit is (1/3)/(1/6) = 2.

Q3. Answer (2). This is a 1^∞ form: the limit is e^L with $L = \lim 3x \times (2/x) = 6$. So the value is e^6.

Q4. Answer (4). Using $\lim (x^n - a^n)/(x - a) = na^{n-1}$ for the numerator and the denominator: $(5 \times 2^4)/(3 \times 2^2) = 80/12 = 20/3$.

Q5. Answer (2). $\sin 3x = 3 \sin x - 4 \sin^3 x$, so the numerator is $-4 \sin^3 x$ and the limit is −4 (since $\sin x/x \to 1$).

Q6. Answer (1). Put $h = \pi/2 - x$: $\sin x = \cos h$, so the expression is $(1 - \cos h)/h^2 = 2 \sin^2(h/2)/h^2 \to 1/2$.

Q7. Answer (3). Rationalising, $f(x) = 2k/(\sqrt{(1 + kx)} + \sqrt{(1 - kx)}) \to 2k/2 = k$ as $x \to 0$. Continuity needs $k = 2k + 1$, so $k = -1$.

Q8. Answer (4). $(1 - \cos ax)/x^2 = 2 \sin^2(ax/2)/x^2 \to 2 \times a^2/4 = a^2/2$. Continuity: $a^2/2 = 8$, so $a = 4$.

Q9. Answer (2). $dy/dx = [1 + x/\sqrt{(x^2 + 1)}]/(x + \sqrt{(x^2 + 1)}) = 1/\sqrt{(x^2 + 1)}$. So $\sqrt{(x^2 + 1)}\, y' = 1$; differentiating, $\sqrt{(x^2 + 1)}\, y'' + x\, y'/\sqrt{(x^2 + 1)} = 0$, i.e. $(x^2 + 1)y'' + xy' = 0$.

Q10. Answer (1). $(\cos x - \sin x)/(\cos x + \sin x) = (1 - \tan x)/(1 + \tan x) = \tan(\pi/4 - x)$, and $\pi/4 - x$ lies in $(0, \pi/4)$. So the function is $\pi/4 - x$, with derivative −1.

Q11. Answer (3). $x^2 + y^2 = 25$ gives $x\, dx/dt + y\, dy/dt = 0$. At $x = 4$, $y = 3$: $4 \times 2 + 3\, dy/dt = 0$, $dy/dt = -8/3$. The top slides down at 8/3 m/s.

Q12. Answer (4). $f'(x) = 3x^2 - 6x - 9 = 3(x - 3)(x + 1)$, which is negative exactly for $-1 < x < 3$.

Q13. Answer (2). $f'(x) = e^{-x}(2x - x^2) = x(2 - x)e^{-x}$ is positive on (0, 2) and negative after 2, so the greatest value is $f(2) = 4/e^2$ ($f(0) = 0$ and $f \to 0$ as $x \to \infty$).

Q14. Answer (1). With height h, the radius r satisfies $r^2 = 3 - h^2/4$. $V = \pi h(3 - h^2/4)$; $V' = \pi(3 - 3h^2/4) = 0$ at $h = 2$, a maximum ($V'' < 0$). So $h = 2$.

Q15. Answer (2). Statement I is true: $|x \sin(1/x)| \le |x| \to 0$. Statement II is false: $\sin(1/x)$ takes every value between −1 and 1 in every interval around 0 (it is 1 at $x = 2/((4n + 1)\pi)$ and −1 at $x = 2/((4n + 3)\pi)$), so it has no limit.

Q16. Answer (3). A is true: $f(x) \ge 0 = f(0)$ for all x. R is false: the left derivative at 0 is −1 and the right derivative is 1, so $f'(0)$ does not exist. (A local extremum does not need a zero derivative.)

Q17. Answer (4). $(a^x - 1)/x \to \ln a$, so the limit is $\ln 2/\ln 3 = \log_3 2$.

Q18. Answer (1). $dy/dx = \sin t/(1 - \cos t) = \cot(t/2)$. $d^2y/dx^2 = [-(1/2)\,\mathrm{cosec}^2(t/2)]/[a(1 - \cos t)]$. At $t = \pi/2$: $\mathrm{cosec}^2(\pi/4) = 2$ and $1 - \cos t = 1$, so $d^2y/dx^2 = -1/a$.

Q19. Answer (3). $f'(x) = 6(x^2 - 5x + 6) = 6(x - 2)(x - 3)$. Compare $f(1) = 24$, $f(2) = 29$, $f(3) = 28$ and $f(5) = 250 - 375 + 180 + 1 = 56$. The greatest is 56.

Q20. Answer (4). With $u = (2x + 3)/(3 - 2x)$: $u(0) = 1$ and $du/dx = 12/(3 - 2x)^2$, which is $4/3$ at $x = 0$. $f'(u) = \cos(\ln u)/u$ is 1 at $u = 1$. So $dy/dx = 1 \times 4/3 = 4/3$.

Q21. Answer 6. $\tan x - \sin x = \tan x\,(1 - \cos x) = \tan x \times 2\sin^2(x/2)$, so $(\tan x - \sin x)/x^3 \to 1 \times 2 \times 1/4 = 1/2$. Then $12L = 6$.

Q22. Answer 27. $f'(x) = 4x^3 - 16x = 4x(x - 2)(x + 2)$. Local maximum at $x = 0$: $f = 5$. Local minima at $x = \pm 2$: $f = 16 - 32 + 5 = -11$ each. Sum of absolute values $5 + 11 + 11 = 27$.

Q23. Answer 54. $dy/dt = 3x^2\,dx/dt = 3 \times 9 \times 2 = 54$.

Q24. Answer 18. At $x = 1$: $a + 1 = 1 + b$, so $a = b$. At $x = 3$: $9 + b = 15 - a$, so $a + b = 6$. Hence $a = b = 3$ and $a^2 + b^2 = 18$.

Q25. Answer 12. $f'(x) = 3x^2 + 2ax + b$ must be $3(x - 1)(x - 5) = 3x^2 - 18x + 15$, so $a = -9$ and $b = 15$. f' changes from + to − at $x = 1$, so the local maximum value is $f(1) = 1 - 9 + 15 + 5 = 12$.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: limits were evaluated numerically from both sides and extrapolated, derivatives found by central differences, monotonicity read from the slope on a fine grid, and maxima and minima found by dense search.