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Matrices and Determinants — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. A matrix is written row by row: [[1, 2], [3, 4]] has rows (1, 2) and (3, 4). I is the identity matrix and O the zero matrix. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Algebra of matrices · Types of matrices · Properties of determinants · Area of a triangle · Adjoint and inverse · Systems of linear equations

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. Let $A = [[1, 2], [3, 4]]$. Then A^{-1} is equal to

(1) $(A - 5I)/2$

(2) $(5I - A)/2$

(3) $(A + 5I)/2$

(4) $(A - 5I)/(-2)$

2. The determinant with rows $(1, a, a^2)$, $(1, b, b^2)$, $(1, c, c^2)$ is equal to

(1) $(a - b)(b - c)(c - a)$

(2) $(a + b)(b + c)(c + a)$

(3) $abc(a - b)(b - c)(c - a)$

(4) $(a - b)(b - c)(a - c)$

3. Statement I: The determinant of every skew-symmetric matrix of odd order is zero.
Statement II: Every skew-symmetric matrix of even order is invertible.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

4. For non-zero a and b, the points $(a, 0)$, $(0, b)$ and $(1, 1)$ are collinear exactly when

(1) $1/a + 1/b = 1$

(2) $a + b = 1$

(3) $1/a + 1/b = 0$

(4) $ab = 1$

5. The system $x + y + z = 3$, $x + 2y + 2z = 5$, $2x + 3y + \lambda z = \mu$ has infinitely many solutions exactly when

(1) $\lambda = 3, \mu \neq 8$

(2) $\lambda \neq 3$

(3) $\lambda = 3, \mu = 8$

(4) $\lambda = 8, \mu = 3$

6. If A and B are symmetric matrices of the same order, then $AB - BA$ is

(1) symmetric

(2) skew-symmetric

(3) the identity matrix

(4) always the zero matrix

7. Let A = [[1, 2, 0], [0, 1, 3], [1, 0, 1]]. The entry in the second row and third column of A^{-1} is

(1) 3/7

(2) −3/7

(3) 6/7

(4) −6/7

8. If A = [[1, 0], [2, 1]], then A^{20} is

(1) [[1, 0], [20, 1]]

(2) [[1, 0], [40, 1]]

(3) [[20, 0], [40, 20]]

(4) [[1, 0], [2^{20}, 1]]

9. The value of the determinant with rows (x + 1, x + 2, x + 3), (x + 2, x + 3, x + 4), (x + 3, x + 4, x + 5) is

(1) −2

(2) x

(3) 0

(4) 1

10. The roots of the equation given by the determinant with rows (x, 2, 3), (2, x, 3), (2, 3, x) equal to zero are

(1) 2, 3, 5

(2) −2, −3, 5

(3) 2, −3, 5

(4) 2, 3, −5

11. Let A = [[cos θ, −sin θ], [sin θ, cos θ]] with θ = π/10. Then A^5 is

(1) [[0, 1], [−1, 0]]

(2) [[1, 0], [0, 1]]

(3) [[0, −1], [1, 0]]

(4) [[−1, 0], [0, −1]]

12. If A is a 3 × 3 matrix with A(adj A) = 5I, then |adj A| is

(1) 5

(2) 125

(3) 25

(4) 1/5

13. The homogeneous system x + y + z = 0, x + 2y + kz = 0, x + 4y + k^2z = 0 has a non-trivial solution exactly when k is

(1) 1 or 2

(2) −1 or −2

(3) 2 or 4

(4) 0 or 1

14. If A is a 3×3 matrix with $|A| = 2$, then $|3A^{-1}A^{T}|$ is

(1) 9

(2) 3

(3) 54

(4) 27

15. If A is a square matrix with $A^2 = A$, then $(I + A)^5$ is equal to

(1) $I + 5A$

(2) $I + 31A$

(3) $I + 32A$

(4) $32I + A$

16. The solution (x, y, z) of $2x - y + z = 3$, $x + y - z = 0$, $x + 2y + z = 4$ is

(1) (1, 1, 1)

(2) (1, 5/3, 2/3)

(3) (2/3, 1, 5/3)

(4) (1, 2/3, 5/3)

17. The number of 2×2 matrices with every entry 0 or 1 whose determinant is zero is

(1) 6

(2) 8

(3) 10

(4) 12

18. Assertion A: For a 3×3 matrix $A = [a_{ij}]$ with cofactors C_{ij}, $a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0$.
Reason R: This sum is the expansion, along its second row, of the determinant obtained from A by replacing the second row by the first, which has two equal rows.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

19. A square matrix that is both symmetric and skew-symmetric is

(1) the identity matrix

(2) a diagonal matrix with equal entries

(3) a zero matrix

(4) any scalar matrix

20. Let A be a 2×2 matrix with trace 4 and determinant 3. Then $|A + I|$ is

(1) 4

(2) 7

(3) 6

(4) 8

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

21. If A is a 2×2 matrix with $|A| = 3$, then $|\text{adj}(\text{adj}(2A))|$ is equal to _____.

Answer: _____

22. Let $A = [[0, 1, 2], [0, 0, 3], [0, 0, 0]]$. The sum of all the entries of $(I + A)^3$ is _____.

Answer: _____

23. Let $A = [[3, 1], [2, 2]]$. If $A^5 = pA + qI$ for real numbers p and q, then $p - q$ is equal to _____.

Answer: _____

24. Let $A = [[2, -1], [1, 3]]$. If $A^{-1} = (pI + qA)/7$, then 7(sum of all the entries of A^{-1}) is equal to _____.

Answer: _____

25. Let $A = [[1, 2, 2], [2, 1, 2], [2, 2, 1]]$. If $A^2 - 4A = kI$, then k is equal to _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(1)	(1)	(2)	(1)	(3)	(2)	(2)	(2)	(3)	(4)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(3)	(3)	(1)	(4)	(2)	(4)	(3)	(1)	(3)	(4)

Q21	Q22	Q23	Q24	Q25
12	30	681	5	5

WORKED SOLUTIONS

Q1. Answer (1). A satisfies its characteristic equation $\lambda^2 - (\text{trace})\lambda + \det = \lambda^2 - 5\lambda - 2 = 0$, so $A^2 - 5A - 2I = O$. Multiplying by A^{-1}: $A - 5I - 2A^{-1} = O$, so $A^{-1} = (A - 5I)/2$.

Q2. Answer (1). Subtract row 1 from rows 2 and 3 and take out $(b - a)$ and $(c - a)$: the determinant becomes $(b - a)(c - a)(c - b) = (a - b)(b - c)(c - a)$.

Q3. Answer (2). I: $A^T = -A$ gives $\det A = \det A^T = (-1)^n \det A$, so for odd n, $\det A = 0$: true. II is false: the zero matrix of order 2 (or [[0, 0], [0, 0]]) is skew-symmetric and not invertible.

Q4. Answer (1). Collinear means the area determinant with rows (a, 0, 1), (0, b, 1), (1, 1, 1) is zero: $a(b - 1) - 0 + 1(0 - b) = ab - a - b = 0$. Dividing by ab: $1/a + 1/b = 1$.

Q5. Answer (3). The coefficient determinant is $\lambda - 3$, so for $\lambda \neq 3$ the solution is unique. For $\lambda = 3$ the third left-hand side is the sum of the first two, so the system is consistent only if $\mu = 3 + 5 = 8$; then there are infinitely many solutions, otherwise none.

Q6. Answer (2). $(AB - BA)^T = B^TA^T - A^TB^T = BA - AB = -(AB - BA)$. It is not always zero: symmetric matrices need not commute.

Q7. Answer (2). $\det A = 1(1 - 0) - 2(0 - 3) + 0 = 7$. $(A^{-1})_{23} = (\text{adj } A)_{23}/\det A = C_{32}/7$, where $C_{32} = (-1)^{3+2} \times \det[[1, 0], [0, 3]] = -3$. So the entry is $-3/7$.

Q8. Answer (2). $A = I + N$ with $N = [[0, 0], [2, 0]]$ and $N^2 = O$, so $A^n = I + nN = [[1, 0], [2n, 1]]$. For n = 20: [[1, 0], [40, 1]].

Q9. Answer (3). $C_3 - C_2$ and $C_2 - C_1$ are both the column (1, 1, 1), so after these operations two columns are equal and the determinant is 0 for every x.

Q10. Answer (4). Expanding: $x(x^2 - 9) - 2(2x - 6) + 3(6 - 2x) = x^3 - 19x + 30 = (x - 2)(x - 3)(x + 5)$. (x = 2 and x = 3 make two rows equal.)

Q11. Answer (3). A is the rotation through θ, and A^n is the rotation through $n\theta$. $5\theta = \pi/2$: $[[\cos \pi/2, -\sin \pi/2], [\sin \pi/2, \cos \pi/2]] = [[0, -1], [1, 0]]$.

Q12. Answer (3). $A(\text{adj } A) = |A| I$, so $|A| = 5$, and $|\text{adj } A| = |A|^{n-1} = 5^2 = 25$.

Q13. Answer (1). A non-trivial solution needs a zero coefficient determinant: $1(2k^2 - 4k) - 1(k^2 - k) + 1(4 - 2) = k^2 - 3k + 2 = (k - 1)(k - 2) = 0$.

Q14. Answer (4). $|3A^{-1}A^T| = 3^3 |A^{-1}| |A^T| = 27 \times (1/2) \times 2 = 27$.

Q15. Answer (2). Since A commutes with I and $A^k = A$ for $k \geq 1$, $(I + A)^5 = I + [C(5, 1) + \ldots + C(5, 5)]A = I + (2^5 - 1)A = I + 31A$.

Q16. Answer (4). Adding the first two equations: $3x = 3$, $x = 1$. Then $y - z = -1$ and $2y + z = 3$ give $3y = 2$, $y = 2/3$, $z = 5/3$. (The coefficient determinant is $9 \neq 0$, so this is the only solution.)

Q17. Answer (3). det $= ad - bc = 0$ needs $ad = bc$. $ad = 0$ in 3 of the 4 choices of (a, d), and likewise $bc = 0$ in 3: 9 matrices; $ad = bc = 1$: 1 more. Total 10.

Q18. Answer (1). Expanding a determinant along row 2 uses C_{21}, C_{22}, C_{23}, which do not depend on row 2 itself. Putting the entries of row 1 in row 2 gives a determinant with two identical rows, i.e. 0. Both true; R explains A. (This is why $A(\text{adj } A) = |A|\, I$.)

Q19. Answer (3). $A^T = A$ and $A^T = -A$ give $A = -A$, so every entry is 0.

Q20. Answer (4). For a 2×2 matrix, $|A + tI| = t^2 + (\text{trace } A)t + |A|$. With $t = 1$: $1 + 4 + 3 = 8$. (Equivalently the eigenvalues are 1 and 3, so those of $A + I$ are 2 and 4.)

Q21. Answer 12. For a 2×2 matrix $B = [[p, q], [r, s]]$, adj $B = [[s, -q], [-r, p]]$, and taking the adjoint again gives back B. So $\text{adj}(\text{adj}(2A)) = 2A$, and $|2A| = 2^2 \times 3 = 12$.

Q22. Answer 30. $A^2 = [[0, 0, 3], [0, 0, 0], [0, 0, 0]]$ and $A^3 = O$, so $(I + A)^3 = I + 3A + 3A^2$. Sum of entries: $3 + 3 \times (1 + 2 + 3) + 3 \times 3 = 3 + 18 + 9 = 30$.

Q23. Answer 681. By the Cayley–Hamilton theorem $A^2 - 5A + 4I = O$, so every power of A has the form $pA + qI$, and p, q follow from the eigenvalues 1 and 4 (the roots of $x^2 - 5x + 4$): $4p + q = 4^5 = 1024$ and $p + q = 1^5 = 1$. So $p = 341$, $q = -340$ and $p - q = 681$.

Q24. Answer 5. $A^2 - 5A + 7I = O$ (trace 5, det 7), so $A^{-1} = (5I - A)/7$. $7A^{-1} = 5I - A = [[3, 1], [-1, 2]]$, whose entries add up to 5.

Q25. Answer 5. Each diagonal entry of A^2 is $1 + 4 + 4 = 9$ and each off-diagonal entry is $2 + 2 + 4 = 8$ (for example row 1 times column 2: $1 \cdot 2 + 2 \cdot 1 + 2 \cdot 2$). So $A^2 - 4A$ has diagonal $9 - 4 = 5$ and off-diagonal $8 - 8 = 0$: $A^2 - 4A = 5I$, $k = 5$.

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