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Permutations and Combinations — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Fundamental principle of counting · Permutations · Permutations with restrictions · Combinations · Distribution and selection

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. The number of four-digit numbers with all digits distinct that are divisible by 5 is

(1) 896

(2) 952

(3) 1008

(4) 504

2. The number of arrangements of all the letters of the word MISSISSIPPI in which no two S's are next to each other is

(1) 34650

(2) 6300

(3) 7350

(4) 8400

3. Twelve points lie in a plane, and exactly 5 of them lie on one straight line; no other three of the points are collinear. The number of triangles with vertices among these points is

(1) 220

(2) 200

(3) 216

(4) 210

4. The number of ways in which 5 boys and 4 girls can stand in a row so that no two girls are next to each other is

(1) 43200

(2) 14400

(3) 86400

(4) 2880

5. Six persons, including A and B, sit around a round table. The number of arrangements in which A and B do NOT sit next to each other is

(1) 48

(2) 72

(3) 96

(4) 120

6. On a 6×6 square grid (like a chessboard with 6 rows and 6 columns), the number of rectangles formed by the grid lines that are NOT squares is

(1) 441

(2) 91

(3) 300

(4) 350

7. A committee of 5 is to be chosen from 6 men and 4 women so that it has at least 2 women. The number of ways is

(1) 192

(2) 180

(3) 186

(4) 246

8. The number of even positive divisors of 10800 is

(1) 48

(2) 60

(3) 45

(4) 36

9. All the arrangements of the letters of the word MASTER are listed in dictionary order. The position (rank) of the word MASTER in this list is

(1) 256

(2) 241

(3) 257

(4) 265

10. If ${}^{20}C_{r+2} = {}^{20}C_{2r-3}$, then the sum of all possible values of r is

(1) 5

(2) 7

(3) 19

(4) 12

11. The number of ordered triples (x, y, z) of positive odd integers with $x + y + z = 15$ is

(1) 36

(2) 28

(3) 21

(4) 45

12. The number of four-digit numbers in which at least one digit is repeated is

(1) 4464

(2) 4536

(3) 5040

(4) 3960

13. The greatest number of points of intersection of 8 straight lines and 4 circles in a plane is

(1) 92

(2) 96

(3) 104

(4) 112

14. Statement I: ${}^{n}P_{r} = r! \times {}^{n}C_{r}$ for $0 \le r \le n$.
Statement II: The number of ways of putting 4 different balls into 3 different boxes (any box may hold any number of balls) is 4^3.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

15. The number of different selections of 4 letters that can be made from the letters of the word EXAMINATION is

(1) 130

(2) 70

(3) 330

(4) 136

16. The number of arrangements of all the letters of the word INDEPENDENCE in which all the vowels come together is

(1) 16800

(2) 3360

(3) 33600

(4) 8400

17. The number of ways of dividing 12 different books into three bundles of 4 books each is

(1) 34650

(2) 5775

(3) 1925

(4) 11550

18. Assertion A: The number of different necklaces that can be made with 5 different beads is 12.
Reason R: Arrangements of n different things in a circle number $(n-1)!$, and a necklace and its mirror image (turned over) are the same, so for $n \ge 3$ the number of necklaces is $(n-1)!/2$.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

19. Five different rings are to be worn on the four fingers of one hand (not the thumb). Any finger may carry any number of rings, and the order of the rings on a finger matters. The number of ways is

(1) 1024

(2) 120

(3) 6720

(4) 3360

20. The number of ways of choosing 3 different numbers from {1, 2, 3, …, 30} so that their sum is divisible by 3 is

(1) 1350

(2) 4060

(3) 1000

(4) 1360

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

21. The number of five-digit numbers divisible by 4 that can be formed with the digits 1, 2, 3, 4, 5, each used exactly once, is _____.

Answer: _____

22. The number of four-digit numbers whose digits add up to 9 is _____.

Answer: _____

23. From 8 married couples, a group of 3 persons is to be chosen so that no husband and wife are both in it. The number of ways is _____.

Answer: _____

24. A path from (0, 0) to (5, 4) moves one unit at a time, either to the right or upwards. The number of such paths that pass through the point (2, 2) is _____.

Answer: _____

25. If ${}^{n}P_{4} = 20 \times {}^{n}P_{2}$, then n is equal to _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(2)	(3)	(4)	(1)	(2)	(4)	(3)	(1)	(3)	(4)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(2)	(1)	(3)	(2)	(4)	(1)	(2)	(1)	(3)	(4)

Q21	Q22	Q23	Q24	Q25
24	165	448	60	7

WORKED SOLUTIONS

Q1. Answer (2). Last digit 0: the other three places take $9 \times 8 \times 7 = 504$ ways. Last digit 5: the first digit cannot be 0 or 5 (8 ways), then 8×7 for the middle: 448. Total $504 + 448 = 952$.

Q2. Answer (3). Arrange the 7 other letters M, I, I, I, I, P, P first: $7!/(4!\,2!) = 105$ ways. They leave 8 gaps, and the four S's go into 4 different gaps: ${}^{8}C_{4} = 70$. Total $105 \times 70 = 7350$.

Q3. Answer (4). Any 3 of the 12 points give ${}^{12}C_{3} = 220$ triples; the ${}^{5}C_{3} = 10$ triples from the collinear five give no triangle. So $220 - 10 = 210$.

Q4. Answer (1). Arrange the boys: $5! = 120$. They make 6 gaps; choose 4 of them and arrange the girls there: ${}^{6}C_{4} \times 4! = 15 \times 24 = 360$. Total $120 \times 360 = 43200$.

Q5. Answer (2). Around a table 6 persons sit in $(6 - 1)! = 120$ ways. With A and B together (one unit, two orders): $2 \times 4! = 48$. So $120 - 48 = 72$.

Q6. Answer (4). A rectangle is fixed by 2 of the 7 horizontal and 2 of the 7 vertical lines: $({}^{7}C_{2})^{2} = 441$. Squares of side k number $(7 - k)^{2}$, so $1 + 4 + 9 + 16 + 25 + 36 = 91$ squares. Rectangles that are not squares: $441 - 91 = 350$.

Q7. Answer (3). All committees ${}^{10}C_{5} = 252$, less those with no woman ${}^{6}C_{5} = 6$ and with exactly one woman $4 \times {}^{6}C_{4} = 60$: $252 - 66 = 186$.

Q8. Answer (1). $10800 = 2^{4} \times 3^{3} \times 5^{2}$. An even divisor $2^{a}3^{b}5^{c}$ needs a = 1, …, 4 (4 ways), b = 0, …, 3 (4 ways), c = 0, 1, 2 (3 ways): $4 \times 4 \times 3 = 48$.

Q9. Answer (3). Words starting with A or E: $2 \times 5! = 240$. After M: A is the smallest letter left (0). After MA: from S, T, E, R the letters E, R come before S: $2 \times 3! = 12$. After MAS: from T, E, R, the letters E, R come before T: $2 \times 2! = 4$. Then E, R are in order. Rank $= 240 + 12 + 4 + 1 = 257$.

Q10. Answer (4). ${}^{n}C_{p} = {}^{n}C_{q}$ when p = q or p + q = n. $r + 2 = 2r - 3$ gives r = 5; $r + 2 + 2r - 3 = 20$ gives r = 7. Both are valid (${}^{20}C_{7} = {}^{20}C_{7}$ and ${}^{20}C_{9} = {}^{20}C_{11}$), so the sum is 12.

Q11. Answer (2). Put $x = 2a + 1$, $y = 2b + 1$, $z = 2c + 1$ with $a, b, c \geq 0$: $a + b + c = 6$, which has ${}^{8}C_{2} = 28$ solutions.

Q12. Answer (1). There are 9000 four-digit numbers; those with all digits distinct number $9 \times 9 \times 8 \times 7 = 4536$. So $9000 - 4536 = 4464$.

Q13. Answer (3). Two lines meet in at most 1 point: ${}^{8}C_{2} = 28$. Two circles in at most 2: $2 \times {}^{4}C_{2} = 12$. A line and a circle in at most 2: $2 \times 8 \times 4 = 64$. Total $28 + 12 + 64 = 104$.

Q14. Answer (2). Statement I is true: an arrangement of r things is a selection of r things followed by one of r! orders. Statement II is false: each ball has 3 choices, so the number is $3^{4} = 81$, not $4^{3} = 64$.

Q15. Answer (4). EXAMINATION has 8 different letters, with A, I and N each twice. Four different letters: ${}^{8}C_{4} = 70$. One repeated pair and two other letters: $3 \times {}^{7}C_{2} = 63$. Two pairs: ${}^{3}C_{2} = 3$. Total $70 + 63 + 3 = 136$.

Q16. Answer (1). The vowels are I, E, E, E, E and the consonants N, N, N, D, D, P, C. Treat the vowels as one block: 8 units with N three times and D twice, $8!/(3!\,2!) = 3360$. Inside the block the vowels can be ordered in $5!/4! = 5$ ways. Total 16800.

Q17. Answer (2). Into three named bundles: $12!/(4!\,4!\,4!) = 34650$. The bundles are not named, so divide by 3!: $34650/6 = 5775$.

Q18. Answer (1). Circular arrangements of 5 beads: $4! = 24$; turning a necklace over pairs each arrangement with its mirror image (different for $n \geq 3$), so $24/2 = 12$. A is true and R explains it.

Q19. Answer (3). Line up the 5 rings and 3 dividers between the fingers: 8 places, of which the dividers (alike) take any 3 and the rings fill the rest in order, $8!/3! = 6720$. (Equivalently the rings are placed one by one: $4 \times 5 \times 6 \times 7 \times 8 = 6720$.)

Q20. Answer (4). Each remainder 0, 1, 2 (mod 3) occurs for 10 numbers. The sum is a multiple of 3 when all three have the same remainder ($3 \times {}^{10}C_{3} = 360$) or all three remainders are different ($10^{3} = 1000$). Total 1360.

Q21. Answer 24. A number is divisible by 4 when its last two digits are. From distinct digits 1–5 these endings are 12, 24, 32 and 52. Each leaves 3 digits for the first three places: $3! = 6$. Total $4 \times 6 = 24$.

Q22. Answer 165. $a + b + c + d = 9$ with $a \geq 1$: put $a = a' + 1$, then $a' + b + c + d = 8$ with all ≥ 0, which has ${}^{11}C_{3} = 165$ solutions (no digit can exceed 9).

Q23. Answer 448. Choose the 3 couples that are represented: ${}^{8}C_{3} = 56$, then one person from each: $2^{3} = 8$. Total 448.

Q24. Answer 60. From (0, 0) to (2, 2): 4 steps with 2 to the right, ${}^{4}C_{2} = 6$. From (2, 2) to (5, 4): 5 steps with 3 to the right, ${}^{5}C_{3} = 10$. Total $6 \times 10 = 60$.

Q25. Answer 7. ${}^{n}P_{4}/{}^{n}P_{2} = (n - 2)(n - 3) = 20$, so $n^{2} - 5n - 14 = 0$, $(n - 7)(n + 2) = 0$, $n = 7$.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: the arrangements, selections, numbers, paths and distributions were listed one by one and counted, without using the formulas of the worked solutions.