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Sequence and Series — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Arithmetic progression · Sum of an AP · Geometric progression · Sum of a GP · Arithmetic and geometric means

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. The pth, qth and rth terms of an AP are a, b and c respectively. Then $a(q - r) + b(r - p) + c(p - q)$ is equal to

(1) 1

(2) 0

(3) pqr

(4) a + b + c

2. The sum of all three-digit numbers that leave remainder 2 when divided by 7 is

(1) 70692

(2) 70399

(3) 71241

(4) 69993

3. The sums of the first n terms of two APs are in the ratio $(3n + 8) : (7n + 15)$ for every n. The ratio of their 12th terms is

(1) 3 : 7

(2) 16 : 7

(3) 7 : 16

(4) 11 : 25

4. The 4th term of a GP is 54 and its 7th term is 1458. The sum of its first 6 terms is

(1) 726

(2) 1456

(3) 728

(4) 364

5. Three numbers in GP have sum 21 and product 216. The sum of their squares is

(1) 189

(2) 147

(3) 161

(4) 252

6. n arithmetic means are inserted between 7 and 49, and their sum is 364. Then n is equal to

(1) 12

(2) 14

(3) 26

(4) 13

7. Three positive geometric means are inserted between 2 and 162. The largest of them is

(1) 54

(2) 18

(3) 81

(4) 48

8. The arithmetic mean and the geometric mean of two positive numbers are 10 and 8 respectively. The numbers are

(1) 12 and 8

(2) 16 and 4

(3) 18 and 2

(4) 15 and 5

9. Statement I: If a, b, c are in AP, then 2^a, 2^b, 2^c are in GP.
Statement II: If the positive numbers a, b, c are in GP, then a + b, b + c, c + a are also in GP.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

10. The sum of an infinite GP with positive common ratio is 20, and the sum of its first two terms is 15. The first term of the GP is

(1) 5

(2) 15

(3) 10

(4) 30

11. For $x > 0$, the least value of $4x + 9/x$ is

(1) 13

(2) 6

(3) 36

(4) 12

12. If a, b, c are positive and $1/(b + c)$, $1/(c + a)$, $1/(a + b)$ are in AP, then a^2, b^2, c^2 are

(1) in GP

(2) in AP

(3) neither in AP nor in GP, in general

(4) in AP only when a = b = c

13. The sum of the first n terms of $0.7 + 0.77 + 0.777 + \ldots$ is

(1) $(7/9)(n - 1 + 10^{-n})$

(2) $(7/81)(9n + 1 - 10^{-n})$

(3) $(7/81)(10n - 1 + 10^{-n})$

(4) $(7/81)(9n - 1 + 10^{-n})$

14. The sum of the first 10 terms of an AP is 4 times the sum of its first 5 terms. The ratio of its first term to its common difference is

(1) 1 : 2

(2) 2 : 1

(3) 1 : 4

(4) 4 : 1

15. The number of terms common to the two APs 3, 7, 11, …, 407 and 2, 9, 16, …, 709 is

(1) 13

(2) 14

(3) 15

(4) 12

16. Assertion A: The product of the first 9 terms of a GP whose 5th term is 2 is 512.
Reason R: In a GP, the product of two terms equidistant from the beginning and from the end is equal to the square of the first term.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

17. If a, b, c are in AP and the positive numbers x, y, z are in GP, then x^(b − c) × y^(c − a) × z^(a − b) is equal to

(1) 0

(2) xyz

(3) 1

(4) −1

18. The sum of all integers from 1 to 200 that are divisible by neither 2 nor 5 is

(1) 8100

(2) 7900

(3) 10000

(4) 8000

19. The 3rd, 5th and 8th terms of a non-constant AP are in GP. The common ratio of this GP is

(1) 2/3

(2) 3/2

(3) 2

(4) 4/3

20. The greatest value of the sum of the first n terms of the AP 40, 37, 34, … is

(1) 280

(2) 294

(3) 273

(4) 287

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

21. The sum of all two-digit numbers that are divisible by 4 or by 6 is _____.

Answer: _____

22. The arithmetic mean of two positive numbers a and b (a > b) is twice their geometric mean. Then a/b, correct to the nearest integer, is _____.

Answer: _____

23. Five integers in AP have sum 25 and product 2520. The largest of them is _____.

Answer: _____

24. The number of terms of the GP 3, 3/2, 3/4, … that must be added to get the sum 3069/512 is _____.

Answer: _____

25. In an AP $a_1, a_2, a_3, \ldots, a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$. Then $a_1 + a_2 + \ldots + a_{24}$ is equal to _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(2)	(1)	(3)	(3)	(1)	(4)	(1)	(2)	(2)	(3)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(4)	(2)	(4)	(1)	(2)	(3)	(3)	(4)	(2)	(4)

Q21	Q22	Q23	Q24	Q25
1566	14	7	10	900

WORKED SOLUTIONS

Q1. Answer (2). With first term A and common difference D: a = A + (p − 1)D, and so on. The A-parts give A[(q − r) + (r − p) + (p − q)] = 0 and the D-parts give D[(p − 1)(q − r) + (q − 1)(r − p) + (r − 1)(p − q)] = 0. So the expression is 0.

Q2. Answer (1). The numbers are 100, 107, …, 996 (100 = 7 × 14 + 2 and 996 = 7 × 142 + 2): an AP with (996 − 100)/7 + 1 = 129 terms. Sum = 129 × (100 + 996)/2 = 129 × 548 = 70692.

Q3. Answer (3). $S_n = (n/2)[2a + (n − 1)d]$, so $S_n/S_n' = [a + ((n − 1)/2)d]/[a' + ((n − 1)/2)d']$. The 12th terms are a + 11d, so take (n − 1)/2 = 11, n = 23: (3 × 23 + 8)/(7 × 23 + 15) = 77/176 = 7 : 16.

Q4. Answer (3). $r^3 = 1458/54 = 27$, so r = 3 and a = 54/27 = 2. $S_6 = 2(3^6 − 1)/(3 − 1) = 729 − 1 = 728$.

Q5. Answer (1). Take them as a/r, a, ar: $a^3 = 216$ gives a = 6. Then 6/r + 6 + 6r = 21, $2r^2 − 5r + 2 = 0$, r = 2 or 1/2: the numbers are 3, 6, 12. Sum of squares 9 + 36 + 144 = 189.

Q6. Answer (4). The n means together with 7 and 49 form an AP, so the means pair up to sum 7 + 49 = 56 each: their sum is n × 56/2 = 28n = 364, n = 13.

Q7. Answer (1). 2, 2r, $2r^2$, $2r^3$, 162 is a GP, so $r^4 = 81$, and the means are positive, so r = 3: 6, 18, 54. The largest is 54.

Q8. Answer (2). a + b = 20 and ab = 64, so they are the roots of $t^2 − 20t + 64 = 0$: t = 16 and 4.

Q9. Answer (2). Statement I: 2b = a + c gives $(2^b)^2 = 2^a × 2^c$, so it is true. Statement II is false: 1, 2, 4 are in GP but 3, 6, 5 are not.

Q10. Answer (3). a/(1 − r) = 20 and a(1 + r) = 15. Dividing: (1 − r)(1 + r) = 15/20, so $r^2 = 1/4$ and r = 1/2. Then a = 20(1 − 1/2) = 10.

Q11. Answer (4). AM ≥ GM for the positive numbers 4x and 9/x: (4x + 9/x)/2 ≥ √36 = 6, so 4x + 9/x ≥ 12, with equality at 4x = 9/x, x = 3/2.

Q12. Answer (2). 2/(c + a) = 1/(b + c) + 1/(a + b) = (a + 2b + c)/[(b + c)(a + b)]. Cross-multiplying: $2(ab + b^2 + ac + bc) = (a + c)(a + 2b + c) = a^2 + 2ab + 2ac + 2bc + c^2$, i.e. $2b^2 = a^2 + c^2$. So a^2, b^2, c^2 are in AP.

Q13. Answer (4). The kth term is $(7/9)(1 − 10^{-k})$. Sum = $(7/9)[n − (1/10)(1 − 10^{-n})/(1 − 1/10)] = (7/9)[n − (1 − 10^{-n})/9] = (7/81)(9n − 1 + 10^{-n})$.

Q14. Answer (1). $S_{10} = 5(2a + 9d)$ and $4S_5 = 4 × (5/2)(2a + 4d) = 10(2a + 4d)$. So 10a + 45d = 20a + 40d, 10a = 5d, a : d = 1 : 2.

Q15. Answer (2). The first AP is the numbers ≡ 3 (mod 4) and the second the numbers ≡ 2 (mod 7). The first common term is 23, and the common terms form the AP 23, 51, 79, … with difference lcm(4, 7) = 28. They must not exceed 407: 23 + 28(k − 1) ≤ 407 gives k ≤ 14.7, so 14 terms.

Q16. Answer (3). A: the terms are ar^{0}, …, ar^{8} with product $a^{9}r^{36} = (ar^{4})^{9} = 2^{9} = 512$, so A is true. R is false: the product of terms equidistant from the two ends equals the product of the first and last terms (here the square of the middle term), not the square of the first term.

Q17. Answer (3). Take $b - c = -d$, $c - a = 2d$, $a - b = -d$ (d the common difference) and $y = xr$, $z = xr^{2}$: $x^{-d}\,(xr)^{2d}\,(xr^{2})^{-d} = x^{-d+2d-d}\,r^{2d-2d} = 1$.

Q18. Answer (4). All: $200 \times 201/2 = 20100$. Multiples of 2: $2(1 + \dots + 100) = 10100$. Multiples of 5: $5(1 + \dots + 40) = 4100$. Multiples of 10 (counted twice): $10(1 + \dots + 20) = 2100$. So $20100 - 10100 - 4100 + 2100 = 8000$.

Q19. Answer (2). $(a + 4d)^{2} = (a + 2d)(a + 7d)$ gives $16d^{2} + 8ad = 14d^{2} + 9ad$, so $a = 2d$ ($d \neq 0$). The terms are $4d$, $6d$, $9d$, with ratio $3/2$.

Q20. Answer (4). The terms $40 - 3(k - 1)$ stay positive up to $k = 14$ (the 14th term is 1, the 15th is -2). The greatest sum is $S_{14} = 14 \times (40 + 1)/2 = 287$.

Q21. Answer 1566. Multiples of 4: 12, 16, …, 96 (22 terms, sum $22 \times 54 = 1188$). Multiples of 6: 12, 18, …, 96 (15 terms, sum $15 \times 54 = 810$). Multiples of 12, counted twice: 12, …, 96 (8 terms, sum $8 \times 54 = 432$). Total $1188 + 810 - 432 = 1566$.

Q22. Answer 14. $(a + b)/2 = 2\sqrt{(ab)}$. With $t = \sqrt{(a/b)}$: $t^{2} - 4t + 1 = 0$, $t = 2 + \sqrt{3}$ ($t > 1$). So $a/b = (2 + \sqrt{3})^{2} = 7 + 4\sqrt{3} \approx 13.93$, which is 14 to the nearest integer.

Q23. Answer 7. Write them $5 - 2d$, $5 - d$, 5, $5 + d$, $5 + 2d$ (the middle one is $25/5 = 5$). Then $5(25 - d^{2})(25 - 4d^{2}) = 2520$, so $4d^{4} - 125d^{2} + 121 = 0$, $d^{2} = 1$ or $121/4$. Integers need $d = \pm 1$: the numbers 3, 4, 5, 6, 7. The largest is 7.

Q24. Answer 10. $S_{n} = 3(1 - (1/2)^{n})/(1 - 1/2) = 6(1 - 2^{-n}) = 3069/512$ gives $1 - 2^{-n} = 3069/3072$, $2^{-n} = 3/3072 = 1/1024$, $n = 10$.

Q25. Answer 900. Terms equidistant from the two ends of $a_{1}, \dots, a_{24}$ have equal sums: $a_{1} + a_{24} = a_{5} + a_{20} = a_{10} + a_{15}$. So $3(a_{1} + a_{24}) = 225$, $a_{1} + a_{24} = 75$, and $S_{24} = 12 \times 75 = 900$.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: the progressions were generated term by term in exact arithmetic and added up directly, conditions were tested on randomly built progressions, and counts and searches ran over the numbers themselves.