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Sets, Relations and Functions — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Sets · Relations · Equivalence relations · Functions · Types of functions · Composition of functions

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. The number of relations on the set $\{1, 2, 3\}$ that are both reflexive and symmetric is

(1) 16

(2) 8

(3) 64

(4) 6

2. The number of equivalence relations on the set $\{1, 2, 3, 4\}$ that contain the pair $(1, 2)$ is

(1) 15

(2) 4

(3) 5

(4) 7

3. The number of onto functions f from $\{1, 2, 3, 4\}$ to $\{a, b, c\}$ with $f(1) = a$ is

(1) 12

(2) 36

(3) 18

(4) 9

4. The number of functions $f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ such that $f(f(x)) = x$ for every x is

(1) 24

(2) 4

(3) 9

(4) 10

5. Which of the following functions from $\mathbb{R}$ to $\mathbb{R}$ is one-one but NOT onto?

(1) $f(x) = x^3 + x$

(2) $f(x) = x^2 + 1$

(3) $f(x) = e^x$

(4) $f(x) = \sin x$

6. Let $f(x) = (x - 1)/(x + 1)$, $x \neq -1$. Then $f(f(f(x)))$, wherever defined, is equal to

(1) x

(2) $-1/x$

(3) $(x + 1)/(1 - x)$

(4) $(1 - x)/(1 + x)$

7. Let $A = \{1, 2, 3, 4, 5, 6\}$. The number of subsets B of A such that $B \cap \{1, 2\} = \{1\}$ is

(1) 32

(2) 16

(3) 8

(4) 64

8. Let $R = \{(a, b) : a, b \in \mathbb{N}, a + 3b = 12\}$. The domain of R is

(1) $\{1, 2, 3\}$

(2) $\{3, 6, 9\}$

(3) $\{3, 6, 9, 12\}$

(4) $\{0, 3, 6, 9\}$

9. The domain of $f(x) = 1/\sqrt{(x^2 - 5x + 6)} + \ln(4 - x)$ is

(1) $(-\infty, 2) \cup (3, 4)$

(2) $(2, 3)$

(3) $(-\infty, 2] \cup [3, 4)$

(4) $(-\infty, 4)$

10. The function $f : [1, \infty) \to [2, \infty)$, $f(x) = x + 1/x$ is a bijection. Its inverse is

(1) $f^{-1}(y) = (y - \sqrt{(y^2 - 4)})/2$

(2) $f^{-1}(y) = (y + \sqrt{(y^2 - 4)})/2$

(3) $f^{-1}(y) = (y + \sqrt{(y^2 + 4)})/2$

(4) $f^{-1}(y) = 1/(y - 1)$

11. The number of strictly increasing functions $f : \{1, 2, 3\} \to \{1, 2, 3, 4, 5\}$ is

(1) 60

(2) 125

(3) 35

(4) 10

12. Let $f : A \to B$ and $g : B \to C$ be functions.
Statement I: If $g \circ f$ is one-one, then f is one-one.
Statement II: If $g \circ f$ is onto, then f is onto.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

13. Let $f(x) = x^2 + 1$, $x \in \mathbb{R}$, and $g(x) = \sqrt{(x - 1)}$, $x \geq 1$. Then $(g \circ f)(x)$ is

(1) x

(2) $|x|$

(3) x^2

(4) $\sqrt{(x^2 + 1)}$

14. The range of $f(x) = (x^2 + x + 2)/(x^2 + x + 1)$, $x \in \mathbb{R}$, is

(1) $(1, 7/3]$

(2) $[1, 7/3]$

(3) $(1, \infty)$

(4) $[7/3, \infty)$

15. For any three sets A, B and C, the set $A - (B \cup C)$ is equal to

(1) $(A - B) \cup (A - C)$

(2) $(A - B) \cap (A - C)$

(3) $(A - B) \cup C$

(4) $(A \cap B) - C$

16. The number of elements in the relation $R = \{(x, y) : x, y \in \mathbb{Z}, x^2 + y^2 \le 4\}$ is

(1) 9

(2) 12

(3) 13

(4) 21

17. The relation R on $\mathbb{R}$ defined by a R b if $|a - b| \le 1$ is

(1) an equivalence relation

(2) reflexive and transitive but not symmetric

(3) reflexive and symmetric but not transitive

(4) symmetric and transitive but not reflexive

18. The function $f : \mathbb{R} \to (-1, 1)$, $f(x) = x/(1 + |x|)$, is

(1) one-one but not onto

(2) onto but not one-one

(3) both one-one and onto

(4) neither one-one nor onto

19. Let $A = \{x \in \mathbb{Z} : |x| < 3\}$ and $B = \{x \in \mathbb{N} : x^2 < 10\}$. The number of elements in $A \times B$ is

(1) 15

(2) 21

(3) 12

(4) 18

20. The function $f(x) = \ln(x + \sqrt{(x^2 + 1)})$, $x \in \mathbb{R}$, is

(1) even

(2) neither even nor odd

(3) both even and odd

(4) odd

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

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21. The number of equivalence relations R on the set $\{1, 2, 3, 4, 5\}$ such that every equivalence class of R has at least two elements is _____.

Answer: _____

22. The number of relations on the set $\{1, 2, 3\}$ that are both symmetric and transitive is _____.

Answer: _____

23. In a group of 100 students, 50 study Mathematics, 40 Physics and 30 Chemistry; 15 study both Mathematics and Physics, 10 both Physics and Chemistry, 12 both Mathematics and Chemistry, and 5 all three. The number of students who study none of the three subjects is _____.

Answer: _____

24. The number of functions from {1, 2, 3, 4} to {1, 2, 3} that are NOT onto is _____.

Answer: _____

25. Let A = {1, 2, 3, 4}. The number of ordered pairs (B, C) of subsets of A such that $B \subseteq C$ is _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(2)	(3)	(1)	(4)	(3)	(3)	(2)	(2)	(1)	(2)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(4)	(2)	(2)	(1)	(2)	(3)	(3)	(3)	(1)	(4)

Q21	Q22	Q23	Q24	Q25
11	15	12	45	81

WORKED SOLUTIONS

Q1. Answer (2). Reflexive: all three pairs (a, a) must be present. Symmetric: the six other pairs come in three pairs {(a, b), (b, a)}, each pair taken together or not at all: $2^3 = 8$ choices.

Q2. Answer (3). An equivalence relation is the same as a partition of the set into classes. If (1, 2) is in it, 1 and 2 are in the same class, so treat them as one element: partitions of a 3-element set, which number $B_3 = 5$.

Q3. Answer (1). f(2), f(3), f(4) must take both b and c among them (a is already taken by f(1)). Maps of 3 elements into {a, b, c} missing b: $2^3 = 8$; missing c: 8; missing both: 1. So $27 - 8 - 8 + 1 = 12$.

Q4. Answer (4). f must be its own inverse, so it is one-one, and it either fixes an element or swaps it with another. All fixed: 1; one swap: C(4, 2) = 6; two swaps: 3. Total 10.

Q5. Answer (3). e^x is strictly increasing (one-one), but its values are only the positive reals, so it is not onto $\mathbb{R}$. $x^3 + x$ is strictly increasing with range $\mathbb{R}$ (bijective); $x^2 + 1$ and sin x are not one-one.

Q6. Answer (3). $f(f(x)) = [(x - 1)/(x + 1) - 1]/[(x - 1)/(x + 1) + 1] = (-2)/(2x) = -1/x$. Then $f(-1/x) = (-1/x - 1)/(-1/x + 1) = (-1 - x)/(x - 1) = (x + 1)/(1 - x)$.

Q7. Answer (2). 1 must be in B and 2 must not; each of 3, 4, 5, 6 may or may not be in B: $2^4 = 16$.

Q8. Answer (2). $a = 12 - 3b$ must be a natural number: b = 1, 2, 3 give a = 9, 6, 3 (b = 4 gives a = 0, not in $\mathbb{N}$). So R = {(9, 1), (6, 2), (3, 3)} and its domain is {3, 6, 9}.

Q9. Answer (1). Need $x^2 - 5x + 6 = (x - 2)(x - 3) > 0$ (strictly, as it is under a root in the denominator): $x < 2$ or $x > 3$; and $4 - x > 0$: $x < 4$. Together: $(-\infty, 2) \cup (3, 4)$.

Q10. Answer (2). $x + 1/x = y$ gives $x^2 - yx + 1 = 0$, $x = (y \pm \sqrt{(y^2 - 4)})/2$. The two roots have product 1, so one is ≥ 1 and the other ≤ 1; for $x \in [1, \infty)$ take the + sign.

Q11. Answer (4). A strictly increasing function is fixed by its set of three values, which is any 3-element subset of {1, …, 5}, listed in increasing order: C(5, 3) = 10.

Q12. Answer (2). I: if $f(x_1) = f(x_2)$ then $g(f(x_1)) = g(f(x_2))$, so $x_1 = x_2$: true. II is false: take A = {1}, B = {1, 2}, C = {1}, f(1) = 1, g(1) = g(2) = 1. Then $g \circ f$ is onto C, but f misses 2. (It is g that must be onto.)

Q13. Answer (2). $(g \circ f)(x) = \sqrt{(x^2 + 1 - 1)} = \sqrt{(x^2)} = |x|$, not x (for $x = -2$ it gives 2).

Q14. Answer (1). $f(x) = 1 + 1/(x^2 + x + 1)$. Since $x^2 + x + 1 = (x + ½)^2 + ¾$ takes every value in $[¾, \infty)$, $1/(x^2 + x + 1)$ takes every value in (0, 4/3]. So the range is (1, 7/3]; the value 1 is never reached.

Q15. Answer (2). $x \in A - (B \cup C)$ means $x \in A$, $x \notin B$ and $x \notin C$, i.e. $x \in A - B$ and $x \in A - C$ (De Morgan's law relative to A).

Q16. Answer (3). $x^2 + y^2 \le 4$: $x = 0$ gives $y = -2 \ldots 2$ (5 pairs); $x = \pm 1$ gives $y = -1, 0, 1$ (3 each, 6 pairs); $x = \pm 2$ gives $y = 0$ (2 pairs). Total 13.

Q17. Answer (3). $|a - a| = 0 \le 1$ (reflexive); $|a - b| = |b - a|$ (symmetric); but $0 \mathrel{R} 1$ and $1 \mathrel{R} 2$ while $|0 - 2| = 2 > 1$, so it is not transitive.

Q18. Answer (3). For $x \ge 0$, $f(x) = x/(1 + x)$ increases from 0 towards 1; for $x < 0$, $f(x) = x/(1 - x)$ increases from -1 towards 0. So f is strictly increasing (one-one) and its values fill $(-1, 1)$ (onto): $y \in [0, 1)$ comes from $x = y/(1 - y)$, $y \in (-1, 0)$ from $x = y/(1 + y)$.

Q19. Answer (1). $A = \{-2, -1, 0, 1, 2\}$ has 5 elements; $B = \{1, 2, 3\}$ has 3. $n(A \times B) = 5 \times 3 = 15$.

Q20. Answer (4). $f(-x) = \ln(\sqrt{(x^2 + 1)} - x) = \ln(1/(\sqrt{(x^2 + 1)} + x)) = -f(x)$, since $(\sqrt{(x^2 + 1)} - x)(\sqrt{(x^2 + 1)} + x) = 1$. So f is odd (and not even, as it is not identically zero).

Q21. Answer 11. Equivalence relations correspond to partitions of the set into classes. With no class of size 1, the class sizes can only be 5 (one way) or 3 + 2 (choose the 2-element class: $C(5, 2) = 10$ ways); 2 + 2 + 1 and the rest contain a singleton. Total 11.

Q22. Answer 15. If R is symmetric and transitive and $(a, b) \in R$, then $(a, a) \in R$. So R is an equivalence relation on the set S of elements that appear in it. Count over S: $\varnothing$ (1), one element (3×1), two elements (3×2), all three (5): $1 + 3 + 6 + 5 = 15$.

Q23. Answer 12. $n(M \cup P \cup C) = 50 + 40 + 30 - 15 - 10 - 12 + 5 = 88$, so $100 - 88 = 12$ study none.

Q24. Answer 45. All functions: $3^4 = 81$. Onto: $81 - 3 \times 2^4 + 3 \times 1^4 = 81 - 48 + 3 = 36$. Not onto: $81 - 36 = 45$.

Q25. Answer 81. For each element there are three possibilities: in neither B nor C, in C only, or in both (it cannot be in B without C). So $3^4 = 81$.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: relations and functions on the small sets were all listed and tested, set identities were tested on random sets, and compositions, inverses, domains and ranges were checked numerically.