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Statistics and Probability — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Measures of central tendency · Dispersion · Probability · Conditional probability and Bayes' theorem · Probability distributions

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. The mean of 10 observations is 15. One observation, 25, was copied wrongly and should be 15. The correct mean is

(1) 15

(2) 14

(3) 16

(4) 13

2. The six observations 2, 3, 5, 9, x, 12 ($x > 0$) have median 6. Their mean is

(1) 6

(2) 13/2

(3) 37/6

(4) 19/3

3. The variance of the first 10 natural numbers is

(1) 33/4

(2) 11/2

(3) 99/10

(4) 55/6

4. One group of 20 observations has mean 10 and standard deviation 2; another group of 30 observations has mean 15 and standard deviation 3. The variance of all 50 observations together is

(1) 6.5

(2) 10.6

(3) 13

(4) 12.5

5. The variance of a set of observations is 4. Each observation is multiplied by −3 and then 5 is added. The new variance is

(1) 9

(2) 36

(3) −12

(4) 12

6. The mean deviation about the mean of 2, 6, 8, 10, 14 is

(1) 3

(2) 4

(3) 8/5

(4) 16/5

7. Five observations have mean 4 and variance 5.2. Three of them are 1, 2 and 6. If the other two are x and y, then $|x - y|$ is equal to

(1) 3

(2) 1

(3) 2

(4) 5

8. Two fair dice are thrown. The probability that the sum of the numbers shown is a prime number is

(1) 1/3

(2) 7/18

(3) 5/12

(4) 1/2

9. Three cards are drawn together from a well-shuffled pack of 52 cards. The probability that all three are kings is

(1) 1/221

(2) 1/5525

(3) 1/1326

(4) 4/5525

10. For events A and B, $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cap B) = 0.2$. The probability that exactly one of A and B occurs is

(1) 0.5

(2) 0.7

(3) 0.3

(4) 0.9

11. A speaks the truth in 75% of cases and B in 80% of cases, independently. The probability that they contradict each other in stating the same fact is

(1) 0.6

(2) 0.15

(3) 0.2

(4) 0.35

12. A fair die is thrown twice. Given that the sum of the two numbers is 8, the probability that at least one of them is 5 is

(1) 1/3

(2) 1/5

(3) 2/5

(4) 1/18

13. Machines A, B and C make 50%, 30% and 20% of the items in a factory, and 2%, 3% and 5% of their items respectively are defective. An item chosen at random is defective. The probability that it was made by C is

(1) 9/29

(2) 10/29

(3) 1/3

(4) 2/5

14. A fair die is thrown twice. X is the number of sixes obtained. The variance of X is

(1) 1/3

(2) 25/36

(3) 5/36

(4) 5/18

15. A random variable X takes the values 1, 2, 3, 4, 5 with $P(X = k) = k/15$. The mean of X is

(1) 3

(2) 10/3

(3) 11/3

(4) 4

16. Statement I: If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, then A and B are independent.
Statement II: Two mutually exclusive events, each of positive probability, are independent.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

17. Assertion A: If 10 is added to each observation of a data set, its variance increases by 10.
Reason R: The variance of a data set does not change when the same constant is added to every observation.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

18. Two different numbers are chosen at random from 1, 2, 3, …, 20. The probability that their product is divisible by 4 is

(1) 1/2

(2) 9/19

(3) 10/19

(4) 21/38

19. A committee of 3 is chosen at random from 5 boys and 3 girls. The probability that it has exactly 2 girls is

(1) 3/8

(2) 5/28

(3) 15/56

(4) 1/56

20. The values 1, 2, 3, 4, 5 occur with frequencies 3, 10, k, 6, 2 respectively, and their mean is 2.8. The median and the mode of the data are respectively

(1) 3 and 2

(2) 2 and 3

(3) 3 and 3

(4) 2 and 2

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

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21. The sum of the squares of the deviations of 1, 2, 3, …, 9 from their mean is _____.

Answer: _____

22. Two fair dice are thrown. If p is the probability that the sum is 7 or 11, then 36p is equal to _____.

Answer: _____

23. A random variable X takes the values 1, 2 and 3 with $P(X = x) = cx^2$. If E(X) is the mean of X, then 7E(X) is equal to _____.

Answer: _____

24. A bag has 4 red and 6 black balls. Three balls are drawn at random without replacement. If p is the probability that exactly two of them are red, then 10p is equal to _____.

Answer: _____

25. If M is the mean deviation about the median of 1, 3, 5, 7, 9, 11, 13, then 7M is equal to _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(2)	(4)	(1)	(3)	(2)	(4)	(1)	(3)	(2)	(1)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(4)	(3)	(2)	(4)	(3)	(2)	(4)	(1)	(3)	(1)

Q21	Q22	Q23	Q24	Q25
60	8	18	3	24

WORKED SOLUTIONS

Q1. Answer (2). The total 150 falls by 25 − 15 = 10 to 140, so the correct mean is 14.

Q2. Answer (4). The median of six values is the mean of the 3rd and 4th in order. If $x \le 5$ the median is at most 5, and if $x \ge 9$ it is $(5 + 9)/2 = 7$. So $5 < x < 9$, the middle pair is 5 and x, and $(5 + x)/2 = 6$ gives $x = 7$. Mean = $(2 + 3 + 5 + 9 + 7 + 12)/6 = 38/6 = 19/3$.

Q3. Answer (1). Variance = $(\Sigma x^2)/n - \bar{x}^2 = 385/10 - (11/2)^2 = 38.5 - 30.25 = 8.25 = 33/4$ (the formula $(n^2 - 1)/12$ gives the same).

Q4. Answer (3). Combined mean = $(200 + 450)/50 = 13$. The sum of squares is $20(4 + 100) + 30(9 + 225) = 2080 + 7020 = 9100$, so the variance is $9100/50 - 13^2 = 182 - 169 = 13$.

Q5. Answer (2). Adding a constant does not change the variance; multiplying by k multiplies it by k^2. New variance = $9 \times 4 = 36$.

Q6. Answer (4). The mean is $40/5 = 8$. The absolute deviations are 6, 2, 0, 2, 6, with sum 16. Mean deviation = 16/5.

Q7. Answer (1). Sum 20, so $x + y = 11$. $\Sigma x^2 = 5(5.2 + 16) = 106$, so $x^2 + y^2 = 106 - 41 = 65$. Then $2xy = 121 - 65 = 56$ and $(x - y)^2 = 65 - 56 = 9$, so $|x - y| = 3$.

Q8. Answer (3). The prime sums are 2, 3, 5, 7 and 11, occurring in $1 + 2 + 4 + 6 + 2 = 15$ of the 36 outcomes: $15/36 = 5/12$.

Q9. Answer (2). ${}^4C_3/{}^{52}C_3 = 4/22100 = 1/5525$.

Q10. Answer (1). $P(\text{exactly one}) = P(A) + P(B) - 2P(A \cap B) = 0.4 + 0.5 - 0.4 = 0.5$.

Q11. Answer (4). They contradict when exactly one tells the truth: $0.75 \times 0.2 + 0.25 \times 0.8 = 0.15 + 0.2 = 0.35$.

Q12. Answer (3). Sum 8: (2, 6), (3, 5), (4, 4), (5, 3), (6, 2), five equally likely outcomes. Two contain a 5, so the probability is 2/5.

Q13. Answer (2). $P(\text{defective}) = 0.5 \times 0.02 + 0.3 \times 0.03 + 0.2 \times 0.05 = 0.010 + 0.009 + 0.010 = 0.029$. By Bayes' theorem $P(C \mid \text{defective}) = 0.010/0.029 = 10/29$.

Q14. Answer (4). $P(X = 0) = 25/36$, $P(X = 1) = 10/36$, $P(X = 2) = 1/36$. $E(X) = 12/36 = 1/3$ and $E(X^2) = 14/36 = 7/18$. $\text{Var}(X) = 7/18 - 1/9 = 5/18$.

Q15. Answer (3). The probabilities add up to $15/15 = 1$. $E(X) = \Sigma k^2/15 = 55/15 = 11/3$.

Q16. Answer (2). Statement I: $P(A)P(B) = 0.2 = P(A \cap B)$, so A and B are independent: true. Statement II is false: for such events $P(A \cap B) = 0$ but $P(A)P(B) > 0$.

Q17. Answer (4). R is true: every deviation from the mean is unchanged when a constant is added. So the variance stays the same, and A is false.

Q18. Answer (1). Of the 20 numbers, 10 are odd, 5 are 2 more than a multiple of 4 (2, 6, 10, 14, 18) and 5 are multiples of 4. The product fails to be a multiple of 4 only when both are odd (C(10, 2) = 45 ways) or one is odd and the other is of the form 4k + 2 ($10 \times 5 = 50$ ways). So $P = 1 - 95/190 = 1/2$.

Q19. Answer (3). ${}^{3}C_{2} \times {}^{5}C_{1}/{}^{8}C_{3} = 3 \times 5/56 = 15/56$.

Q20. Answer (1). Mean: $(3 + 20 + 3k + 24 + 10)/(21 + k) = 2.8$ gives $57 + 3k = 58.8 + 2.8k$, so $k = 9$ and $N = 30$. The highest frequency is 10, for the value 2, so the mode is 2. The cumulative frequencies are 3, 13, 22, …, so the 15th and 16th values are both 3 and the median is 3.

Q21. Answer 60. The mean is 5; the deviations are −4, …, 4, and the sum of their squares is $2(1 + 4 + 9 + 16) = 60$.

Q22. Answer 8. Sum 7 occurs in 6 ways and sum 11 in 2 ways: $p = 8/36$, so $36p = 8$.

Q23. Answer 18. $c(1 + 4 + 9) = 1$ gives $c = 1/14$. $E(X) = (1 + 8 + 27)/14 = 18/7$, so $7E(X) = 18$.

Q24. Answer 3. $p = {}^{4}C_{2} \times {}^{6}C_{1}/{}^{10}C_{3} = 36/120 = 3/10$, so $10p = 3$.

Q25. Answer 24. The median is 7. The absolute deviations are 6, 4, 2, 0, 2, 4, 6, with sum 24. $M = 24/7$, so $7M = 24$.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer in exact arithmetic: statistics were computed from the data (or from data built to meet the conditions), and probabilities by listing and counting the whole sample space.