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Three Dimensional Geometry — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Distance and section formula · Direction cosines and ratios · Equation of a line · Angle between lines · Shortest distance

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. The point on the x-axis that is equidistant from the points (1, 2, 3) and (3, 5, −2) is

(1) (−6, 0, 0)

(2) (6, 0, 0)

(3) (4, 0, 0)

(4) (3, 0, 0)

2. The ratio in which the yz-plane divides the segment joining (−2, 4, 7) and (3, −5, 8) is

(1) 2 : 3

(2) 3 : 2

(3) 1 : 2

(4) 4 : 3

3. A line makes angles α, β and γ with the positive x-, y- and z-axes. Then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to

(1) 1

(2) 3

(3) 2

(4) 0

4. A line makes an angle of 45° with the x-axis and 60° with the y-axis. The angle it makes with the z-axis is

(1) 60° or 120°

(2) 30° or 150°

(3) 45°

(4) 90°

5. The direction cosines of the line joining (1, −2, 3) to (3, 4, 6), in that direction, are

(1) (2/7, 3/7, 6/7)

(2) (1/7, 3/7, 6/7)

(3) (−2/7, 6/7, 3/7)

(4) (2/7, 6/7, 3/7)

6. The angle between two lines with direction ratios (1, 1, 2) and $(\sqrt{3}-1, -\sqrt{3}-1, 4)$ is

(1) $\pi/6$

(2) $\pi/3$

(3) $\pi/4$

(4) $\pi/2$

7. The line through the point (1, 2, −1) parallel to the line $(x-3)/2 = (y+1)/(-1) = z/3$ is

(1) $(x-1)/2 = (y-2)/1 = (z+1)/3$

(2) $(x-3)/1 = (y+1)/2 = z/(-1)$

(3) $(x-1)/2 = (y-2)/(-1) = (z+1)/3$

(4) $(x+1)/2 = (y+2)/(-1) = (z-1)/3$

8. The lines $(x-1)/(-3) = (y-2)/(2k) = (z-3)/2$ and $(x-1)/(3k) = (y-1)/1 = (z-6)/(-5)$ are perpendicular. Then k is equal to

(1) 10/7

(2) −7/10

(3) 7/10

(4) −10/7

9. The foot of the perpendicular from the point (2, 3, 4) to the line $x = y = z$ is

(1) (3, 3, 3)

(2) (2, 2, 2)

(3) (4, 4, 4)

(4) (1, 1, 1)

10. The shortest distance between the lines $\vec{r} = \lambda(\hat{i} + \hat{j})$ and $\vec{r} = (\hat{i} + 2\hat{k}) + \mu(\hat{j} + \hat{k})$ is

(1) 3

(2) $\sqrt{3}$

(3) $1/\sqrt{3}$

(4) $\sqrt{6}$

11. The lines $(x-1)/2 = (y+1)/3 = z/4$ and $(x-3)/1 = (y-k)/2 = z/1$ intersect. Then k is equal to

(1) 2

(2) −4

(3) 9/2

(4) 4

12. The distance of the point (2, −1, 3) from the line through the origin with direction ratios (1, 2, 2) is

(1) $\sqrt{14}$

(2) 2

(3) $\sqrt{10}$

(4) $\sqrt{6}$

13. The distance between the parallel lines $\vec{r} = \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 3\hat{i} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$ is

(1) $2\sqrt{2}$

(2) 3

(3) $\sqrt{2}$

(4) 6

14. The points A(2, 3, −4), B(1, −2, 3) and C(3, 8, −11) are collinear. The point A divides BC in the ratio

(1) 2 : 1 internally

(2) 1 : 1 internally

(3) 1 : 2 externally

(4) 1 : 1 externally

15. The centroid of the triangle with vertices (2, 1, 3), (4, −1, 2) and (0, 6, 4) is

(1) (2, 6, 3)

(2) (3, 2, 2)

(3) (2, 2, 3)

(4) (6, 6, 9)

16. Statement I: The lines $x/2 = y/2 = z/1$ and $x/1 = y/(-1) = z/0$ are perpendicular.
Statement II: The points (1, 2, 3), (2, 3, 4) and (4, 5, 7) are collinear.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

17. Assertion A: The lines $\vec{r} = \lambda(\hat{i} + 2\hat{j})$ and $\vec{r} = \hat{k} + \mu(2\hat{i} + \hat{j})$ do not intersect.
Reason R: Two lines in space that are not parallel always intersect.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

18. The line through the points (1, 2, 3) and (4, −1, −3) crosses the xy-plane at

(1) (5/2, −1/2, 0)

(2) (1/2, 5/2, 0)

(3) (3, 1, 0)

(4) (5/2, 1/2, 0)

19. The acute angle between the lines $x = y = z$ and $x = -y = z$ is

(1) $\cos^{-1}(1/\sqrt{3})$

(2) $\cos^{-1}(2/3)$

(3) $\cos^{-1}(1/3)$

(4) $\pi/2$

20. The image of the point (1, 2, 3) in the line $x = y = z$ is

(1) (2, 2, 2)

(2) (1, 2, 3)

(3) (3, 1, 2)

(4) (3, 2, 1)

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

21. The distance between the points (1, −2, 4) and (3, 1, 10) is _____.

Answer: _____

22. The number of distinct lines through the origin that make equal angles with the three coordinate axes is _____.

Answer: _____

23. If d is the shortest distance between the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k})$, then $59d^2$ is equal to _____.

Answer: _____

24. The square of the distance of the point (3, 5, 1) from the line $(x - 1)/1 = (y - 1)/0 = (z - 1)/(-1)$ is _____.

Answer: _____

25. If θ is the acute angle between two diagonals of a cube (joining opposite vertices), then $1/\cos\theta$ is equal to _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(2)	(1)	(3)	(1)	(4)	(2)	(3)	(4)	(1)	(2)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(4)	(3)	(1)	(2)	(3)	(2)	(3)	(4)	(3)	(4)

Q21	Q22	Q23	Q24	Q25
7	4	100	18	3

WORKED SOLUTIONS

Q1. Answer (2). $(x-1)^2+4+9=(x-3)^2+25+4$ gives $-2x+14=-6x+38$, so $x=6$: the point (6, 0, 0).

Q2. Answer (1). A point dividing it in the ratio $k:1$ has x-coordinate $(3k-2)/(k+1)$. On the yz-plane $x=0$, so $k=2/3$: the ratio is 2 : 3 (internally).

Q3. Answer (3). $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$, so $\sin^2\alpha+\sin^2\beta+\sin^2\gamma=3-1=2$.

Q4. Answer (1). $\cos^2\gamma=1-\cos^2 45°-\cos^2 60°=1-1/2-1/4=1/4$, so $\cos\gamma=\pm 1/2$ and $\gamma=60°$ or $120°$.

Q5. Answer (4). Direction ratios $(3-1, 4+2, 6-3)=(2, 6, 3)$, of length $\sqrt{(4+36+9)}=7$. Direction cosines (2/7, 6/7, 3/7).

Q6. Answer (2). Dot product $(\sqrt{3}-1)-(\sqrt{3}+1)+8=6$. Lengths $\sqrt{6}$ and $\sqrt{[(4-2\sqrt{3})+(4+2\sqrt{3})+16]}=\sqrt{24}$. $\cos\theta=6/\sqrt{144}=1/2$, $\theta=\pi/3$.

Q7. Answer (3). A parallel line has the same direction ratios (2, −1, 3); through (1, 2, −1) it is $(x-1)/2=(y-2)/(-1)=(z+1)/3$.

Q8. Answer (4). Perpendicular when $(-3)(3k)+(2k)(1)+(2)(-5)=0$: $-7k-10=0$, $k=-10/7$.

Q9. Answer (1). A point of the line is (t, t, t). $(t-2, t-3, t-4)$ is perpendicular to (1, 1, 1) when $3t-9=0$, $t=3$: the foot is (3, 3, 3).

Q10. Answer (2). $\vec{n}=(\hat{i}+\hat{j})\times(\hat{j}+\hat{k})=\hat{i}-\hat{j}+\hat{k}$, $|\vec{n}|=\sqrt{3}$. With $\vec{a}_2-\vec{a}_1=\hat{i}+2\hat{k}$: $d=|(\vec{a}_2-\vec{a}_1)\cdot\vec{n}|/|\vec{n}|=|1+2|/\sqrt{3}=\sqrt{3}$.

Q11. Answer (4). Points $(1+2s, -1+3s, 4s)$ and $(3+t, k+2t, t)$. z gives $t=4s$; x gives $1+2s=3+4s$, so $s=-1$, $t=-4$. y: $-4=k-8$, so $k=4$.

Q12. Answer (3). The component of $\vec{p}=(2,-1,3)$ along the unit vector $(1,2,2)/3$ is $(2-2+6)/3=2$. So the distance is $\sqrt{(|\vec{p}|^2-2^2)}=\sqrt{(14-4)}=\sqrt{10}$.

Q13. Answer (1). $d=|(\vec{a}_2-\vec{a}_1)\times\vec{b}|/|\vec{b}|=|3\hat{i}\times(\hat{i}+2\hat{j}+2\hat{k})|/3=|-6\hat{j}+6\hat{k}|/3=6\sqrt{2}/3=2\sqrt{2}$.

Q14. Answer (2). The mid-point of BC is $((1+3)/2, (-2+8)/2, (3-11)/2)=(2, 3, -4)=A$. So A divides BC in the ratio 1 : 1 internally.

Q15. Answer (3). Average the coordinates: $((2+4+0)/3, (1-1+6)/3, (3+2+4)/3)=(2, 2, 3)$.

Q16. Answer (2). Statement I: $(2)(1)+(2)(-1)+(1)(0)=0$, so the lines are perpendicular: true. Statement II: the vectors (1, 1, 1) and (3, 3, 4) from the first point are not parallel, so the points are not collinear: false.

Q17. Answer (3). A: a common point would need $(\lambda, 2\lambda, 0)=(2\mu, \mu, 1)$, which is impossible since $0\neq 1$. So A is true. R is false: these two lines are not parallel and still do not meet (they are skew).

Q18. Answer (4). Points of the line: $(1+3t, 2-3t, 3-6t)$. $z=0$ at $t=1/2$, giving (5/2, 1/2, 0).

Q19. Answer (3). Direction ratios (1, 1, 1) and (1, −1, 1): $\cos\theta = (1 - 1 + 1)/(\sqrt{3} \times \sqrt{3}) = 1/3$.

Q20. Answer (4). The foot of the perpendicular is (t, t, t) with $3t - 6 = 0$, so (2, 2, 2). The image is $2(2, 2, 2) - (1, 2, 3) = (3, 2, 1)$.

Q21. Answer 7. $\sqrt{(2^2 + 3^2 + 6^2)} = \sqrt{49} = 7$.

Q22. Answer 4. Equal angles need $|l| = |m| = |n| = 1/\sqrt{3}$. The direction cosines are $(\pm 1, \pm 1, \pm 1)/\sqrt{3}$: 8 choices, and a line has two opposite directions, so there are $8/2 = 4$ lines.

Q23. Answer 100. $n = (2, -1, 1) \times (3, -5, 2) = (3, -1, -7)$, $|n|^2 = 59$. $a_2 - a_1 = (1, 0, -1)$, and $(a_2 - a_1)\cdot n = 3 + 7 = 10$. So $d = 10/\sqrt{59}$ and $59d^2 = 100$.

Q24. Answer 18. Points of the line: $(1 + t, 1, 1 - t)$. The foot from (3, 5, 1): $(t - 2, -4, -t)\cdot(1, 0, -1) = 2t - 2 = 0$, $t = 1$, foot (2, 1, 0). Distance$^2 = 1 + 16 + 1 = 18$.

Q25. Answer 3. For the unit cube the diagonals (0, 0, 0)–(1, 1, 1) and (1, 0, 0)–(0, 1, 1) have directions (1, 1, 1) and (−1, 1, 1): $\cos\theta = (-1 + 1 + 1)/3 = 1/3$, and every pair of diagonals gives $\pm 1/3$, so the acute angle has $\cos\theta = 1/3$. So $1/\cos\theta = 3$.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: points were found by solving the conditions exactly, feet of perpendiculars and shortest distances by direct numerical minimisation of the distance, and every line option was tested by its point and direction.