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Trigonometry — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Trigonometric identities · Trigonometric functions · Inverse trigonometric functions

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. The value of $\sin 20^\circ \sin 40^\circ \sin 60^\circ \sin 80^\circ$ is

(1) 1/16

(2) 3/8

(3) 3/16

(4) $\sqrt{3}/16$

2. The value of $\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$ is

(1) 4

(2) 2

(3) 0

(4) 1

3. The value of $(1 + \cos \pi/8)(1 + \cos 3\pi/8)(1 + \cos 5\pi/8)(1 + \cos 7\pi/8)$ is

(1) 1/4

(2) 1/2

(3) 1/16

(4) 1/8

4. The greatest value of $3 \sin x + 4 \cos x + 2$ is

(1) 5

(2) 7

(3) 9

(4) 6

5. The least value of $9 \tan^2 x + 4 \cot^2 x$ is

(1) 13

(2) 6

(3) 12

(4) 36

6. The fundamental period of $f(x) = |\sin x| + |\cos x|$ is

(1) $\pi/2$

(2) π

(3) 2π

(4) $\pi/4$

7. If A and B are acute angles with $\tan A = 1/2$ and $\tan B = 1/3$, then $A + B$ is equal to

(1) $\pi/6$

(2) $\pi/3$

(3) $\pi/2$

(4) $\pi/4$

8. The value of $\cos^2(\pi/8) + \cos^2(3\pi/8) + \cos^2(5\pi/8) + \cos^2(7\pi/8)$ is

(1) 1

(2) 2

(3) 3/2

(4) 4

9. If $\sin\theta + \cos\theta = 1/5$ and $0 < \theta < \pi$, then $\tan\theta$ is equal to

(1) $-4/3$

(2) $-3/4$

(3) 4/3

(4) 3/4

10. The value of $\sin^{-1}(3/5) + \sin^{-1}(8/17)$ is

(1) $\sin^{-1}(13/85)$

(2) $\cos^{-1}(77/85)$

(3) $\sin^{-1}(77/85)$

(4) $\sin^{-1}(36/85)$

11. The value of $\tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$ is

(1) $\pi/2$

(2) $3\pi/4$

(3) 0

(4) π

12. The value of $\cos(2\sin^{-1}(1/3))$ is

(1) 2/9

(2) 7/9

(3) 1/3

(4) 8/9

13. The set of all solutions of $\tan^{-1}(2x) + \tan^{-1}(3x) = \pi/4$ is

(1) $\{1/6\}$

(2) $\{-1, 1/6\}$

(3) $\{-1\}$

(4) $\{1/6, 1\}$

14. The range of $f(x) = \sin^{-1}x + \tan^{-1}x + \cos^{-1}x$ is

(1) $[0, \pi]$

(2) $[\pi/2, \pi]$

(3) $[\pi/4, 3\pi/4]$

(4) $(\pi/4, 3\pi/4)$

15. For $0 < x < 1$, $\sin(\cot^{-1}(\tan(\cos^{-1}x)))$ is equal to

(1) $\sqrt{(1 - x^2)}$

(2) $1/x$

(3) $x/\sqrt{(1 + x^2)}$

(4) x

16. Statement I: $\cos 1 > \cos 2$ (angles in radians).
Statement II: $\tan 1 > 1$ (angle in radians).
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

17. Assertion A: $\tan^{-1}x + \cot^{-1}x = \pi/2$ for every real x.
Reason R: $\cot^{-1}x = \tan^{-1}(1/x)$ for every $x \neq 0$.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

18. The number of real solutions of $\sin x = x/10$ is

(1) 5

(2) 7

(3) 6

(4) 3

19. If $\cos\theta = -3/5$ and $\pi < \theta < 3\pi/2$, then $\tan(\theta/2)$ is equal to

(1) 2

(2) $-1/2$

(3) $1/2$

(4) -2

20. Wherever defined, $\sin 3\theta/\sin\theta - \cos 3\theta/\cos\theta$ is equal to

(1) 1

(2) 2

(3) 0

(4) 3

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

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21. The greatest value of $5\sin^2x + 12\sin x\cos x$ is _____.

Answer: _____

22. The number of values of x in $[0, 2\pi]$ for which $\sin 2x = \cos x$ is _____.

Answer: _____

23. Positive numbers x, y, z satisfy $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$ and $x + y + z = 6$. Then xyz is equal to _____.

Answer: _____

24. The value of $\sec^2(\tan^{-1}2) + \operatorname{cosec}^2(\cot^{-1}3)$ is _____.

Answer: _____

25. If $3 \sin x + 4 \cos x = 5$, then $100 \tan x$ is equal to _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(3)	(1)	(4)	(2)	(3)	(1)	(4)	(2)	(1)	(3)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(4)	(2)	(1)	(3)	(4)	(1)	(3)	(2)	(4)	(2)

Q21	Q22	Q23	Q24	Q25
9	4	6	15	75

WORKED SOLUTIONS

Q1. Answer (3). $\sin\theta \sin(60° - \theta) \sin(60° + \theta) = (1/4) \sin 3\theta$. With $\theta = 20°$: $\sin 20° \sin 40° \sin 80° = (1/4) \sin 60° = \sqrt{3}/8$. Multiplying by $\sin 60° = \sqrt{3}/2$ gives 3/16.

Q2. Answer (1). $\tan\theta + \cot\theta = 2/\sin 2\theta$. The expression is $(\tan 9° + \cot 9°) - (\tan 27° + \cot 27°) = 2/\sin 18° - 2/\sin 54° = 2(\sin 54° - \sin 18°)/(\sin 18° \sin 54°)$. $\sin 54° - \sin 18° = 1/2$ and $\sin 18° \sin 54° = 1/4$, so the value is 4.

Q3. Answer (4). $\cos 7\pi/8 = -\cos \pi/8$ and $\cos 5\pi/8 = -\cos 3\pi/8$, so the product is $(1 - \cos^2\pi/8)(1 - \cos^2 3\pi/8) = \sin^2(\pi/8) \cos^2(\pi/8) = (1/4) \sin^2(\pi/4) = 1/8$.

Q4. Answer (2). $a \sin x + b \cos x$ ranges over $[-\sqrt{(a^2 + b^2)}, \sqrt{(a^2 + b^2)}] = [-5, 5]$. So the greatest value is $5 + 2 = 7$.

Q5. Answer (3). By AM ≥ GM, $9 \tan^2 x + 4 \cot^2 x \geq 2\sqrt{(36 \tan^2 x \cot^2 x)} = 12$, with equality when $\tan^2 x = 2/3$.

Q6. Answer (1). $f(x + \pi/2) = |\cos x| + |\sin x| = f(x)$, and no smaller positive number works ($f(0) = 1$, while f is larger in between, e.g. $f(\pi/4) = \sqrt{2}$). So the period is $\pi/2$.

Q7. Answer (4). $\tan(A + B) = (1/2 + 1/3)/(1 - 1/6) = 1$, and $A + B$ lies in $(0, \pi)$, so $A + B = \pi/4$.

Q8. Answer (2). $\cos(3\pi/8) = \sin(\pi/8)$, $\cos^2(5\pi/8) = \cos^2(3\pi/8)$ and $\cos^2(7\pi/8) = \cos^2(\pi/8)$. The sum is $2[\cos^2(\pi/8) + \sin^2(\pi/8)] = 2$.

Q9. Answer (1). Squaring: $1 + 2 \sin\theta \cos\theta = 1/25$, so $\sin\theta \cos\theta = -12/25 < 0$ and θ is obtuse. $\sin\theta$ and $\cos\theta$ are the roots of $t^2 - t/5 - 12/25 = 0$: 4/5 and −3/5. So $\tan\theta = -4/3$.

Q10. Answer (3). With $\sin\alpha = 3/5$, $\cos\alpha = 4/5$ and $\sin\beta = 8/17$, $\cos\beta = 15/17$: $\sin(\alpha + \beta) = (3/5)(15/17) + (4/5)(8/17) = 77/85$. $\alpha + \beta$ is acute ($\cos(\alpha + \beta) = 36/85 > 0$), so the sum is $\sin^{-1}(77/85)$.

Q11. Answer (4). $\tan^{-1}2 + \tan^{-1}3 = \pi + \tan^{-1}[(2 + 3)/(1 - 6)] = \pi - \pi/4 = 3\pi/4$ (the product $6 > 1$, both positive). Adding $\tan^{-1}1 = \pi/4$ gives π.

Q12. Answer (2). $\cos 2\theta = 1 - 2 \sin^2\theta$ with $\sin\theta = 1/3$: $1 - 2/9 = 7/9$.

Q13. Answer (1). Taking tan: $5x/(1 - 6x^2) = 1$, so $6x^2 + 5x - 1 = 0$, $x = 1/6$ or -1. For $x = -1$ the left side is $\tan^{-1}(-2) + \tan^{-1}(-3) = -3\pi/4$, not $\pi/4$. So only $x = 1/6$.

Q14. Answer (3). The domain is $[-1, 1]$, where $\sin^{-1}x + \cos^{-1}x = \pi/2$. So $f(x) = \pi/2 + \tan^{-1}x$, which increases from $\pi/2 - \pi/4$ to $\pi/2 + \pi/4$: the range is $[\pi/4, 3\pi/4]$.

Q15. Answer (4). With $\theta = \cos^{-1}x \in (0, \pi/2)$: $\tan\theta = \sqrt{(1 - x^2)}/x$. $\cot^{-1}$ of this is the acute angle φ with $\cot\varphi = \sqrt{(1 - x^2)}/x$, i.e. $\varphi = \pi/2 - \theta$. So $\sin\varphi = \cos\theta = x$.

Q16. Answer (1). Statement I: cos decreases on $[0, \pi]$ and $1 < 2$, so $\cos 1 > \cos 2$: true. Statement II: 1 radian $> \pi/4$ and tan increases on $(0, \pi/2)$, so $\tan 1 > \tan(\pi/4) = 1$: true.

Q17. Answer (3). A is true for all real x ($\cot^{-1}x$ is the angle in $(0, \pi)$ whose cotangent is x, and $\pi/2 - \tan^{-1}x$ is that angle). R is false for $x < 0$: $\cot^{-1}(-1) = 3\pi/4$ but $\tan^{-1}(-1) = -\pi/4$.

Q18. Answer (2). Solutions need $|x| \le 10 < 4\pi$. For $x \ge 0$ the line $y = x/10$ meets $y = \sin x$ at $x = 0$, once more in $(0, \pi)$, and twice in $(2\pi, 3\pi)$ (where $\sin x$ reaches 1 above the line); on $(\pi, 2\pi)$ and on $(3\pi, 10]$ $\sin x < 0 < x/10$, so there are no more. That gives 3 positive solutions, 3 negative ones by symmetry, and $x = 0$: 7 in all.

Q19. Answer (4). $\tan^2(\theta/2) = (1 - \cos\theta)/(1 + \cos\theta) = (8/5)/(2/5) = 4$. $\theta/2$ lies in $(\pi/2, 3\pi/4)$, where tan is negative, so $\tan(\theta/2) = -2$.

Q20. Answer (2). $(\sin 3\theta \cos\theta - \cos 3\theta \sin\theta)/(\sin\theta \cos\theta) = \sin 2\theta/(\sin\theta \cos\theta) = 2$.

Q21. Answer 9. $5\sin^2x + 12 \sin x \cos x = 5/2 - (5/2)\cos 2x + 6 \sin 2x$. The greatest value is $5/2 + \sqrt{(25/4 + 36)} = 5/2 + 13/2 = 9$.

Q22. Answer 4. $\cos x (2 \sin x - 1) = 0$: $\cos x = 0$ gives $\pi/2, 3\pi/2$; $\sin x = 1/2$ gives $\pi/6, 5\pi/6$. So 4 values.

Q23. Answer 6. tan of both sides: $(x + y + z - xyz)/(1 - xy - yz - zx) = \tan\pi = 0$, so $x + y + z = xyz = 6$. (For example $x = 1$, $y = 2$, $z = 3$.)

Q24. Answer 15. $\sec^2(\tan^{-1}2) = 1 + 2^2 = 5$ and $\operatorname{cosec}^2(\cot^{-1}3) = 1 + 3^2 = 10$. Sum 15.

Q25. Answer 75. $3 \sin x + 4 \cos x = 5 \sin(x + \alpha)$ with $\cos\alpha = 3/5$, $\sin\alpha = 4/5$. It equals 5 only when $\sin x = 3/5$ and $\cos x = 4/5$, so $\tan x = 3/4$ and $100 \tan x = 75$.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: expressions were evaluated numerically against each option, ranges and periods found by dense sampling, equations solved by locating sign changes, and identities tested at many random angles.