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Vector Algebra — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · MATHEMATICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Vectors and components · Scalar product · Vector product

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. If $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$, the unit vector in the direction of $\vec{a} + \vec{b}$ is

(1) $(\hat{i} + 5\hat{k})/26$

(2) $(\hat{i} - 5\hat{k})/\sqrt{26}$

(3) $(\hat{i} + 5\hat{k})/\sqrt{26}$

(4) $(\hat{i} + \hat{j} + 5\hat{k})/\sqrt{27}$

2. P and Q have position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$. The position vector of the point that divides PQ externally in the ratio 2 : 1 is

(1) $-3\hat{i} + 3\hat{k}$

(2) $3\hat{i} + 3\hat{j} - 3\hat{k}$

(3) $-(1/3)\hat{i} + (4/3)\hat{j} + (1/3)\hat{k}$

(4) $3\hat{i} - 3\hat{k}$

3. The vectors $2\hat{i} + 3\hat{j} + 6\hat{k}$ and $\lambda\hat{i} + 6\hat{j} + \mu\hat{k}$ are parallel. Then $\lambda + \mu$ is equal to

(1) 12

(2) 8

(3) 18

(4) 16

4. If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and the angle between $\vec{a}$ and $\vec{b}$ is $2\pi/3$, then $|\vec{a} + 2\vec{b}|$ is equal to

(1) $\sqrt{109}$

(2) $\sqrt{79}$

(3) $\sqrt{139}$

(4) 7

5. Vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy $\vec{a} + \vec{b} + \vec{c} = 0$, with $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $|\vec{c}| = 7$. The angle between $\vec{a}$ and $\vec{b}$ is

(1) $\pi/3$

(2) $\pi/6$

(3) $2\pi/3$

(4) $\pi/4$

6. The vector component of $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ along $\vec{b} = \hat{i} + \hat{j}$ is

(1) $3(\hat{i} + \hat{j})$

(2) $(3/\sqrt{2})(\hat{i} + \hat{j})$

(3) $(3/2)(\hat{i} + \hat{j})$

(4) $(1/2)(\hat{i} + \hat{j})$

7. The diagonals of a parallelogram are $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$. Its area is

(1) $10\sqrt{3}$

(2) $5\sqrt{2}$

(3) 15

(4) $5\sqrt{3}$

8. If $a \times b = c \times d$ and $a \times c = b \times d$, then $(a - d) \times (b - c)$ is equal to

(1) the zero vector

(2) $a \times d$

(3) $2(a \times b)$

(4) $b \times c$

9. If $|a \times b|^2 + (a \cdot b)^2 = 144$ and $|a| = 4$, then $|b|$ is equal to

(1) 9

(2) 3

(3) 12

(4) $\sqrt{3}$

10. If a, b and c are unit vectors with $a + b + c = 0$, then $a \cdot b + b \cdot c + c \cdot a$ is equal to

(1) 3/2

(2) 0

(3) $-3/2$

(4) -3

11. For unit vectors a and b, $|a + b| = \sqrt{3}\,|a - b|$. The angle between a and b is

(1) $\pi/6$

(2) $\pi/4$

(3) $2\pi/3$

(4) $\pi/3$

12. A force $F = 2\hat{i} + \hat{j} + \hat{k}$ moves a particle from the point (1, 1, 1) to the point (3, 4, 2). The work done is

(1) 6

(2) 8

(3) 9

(4) 12

13. If $a = \hat{i} - \hat{j} + 2\hat{k}$ and $b = 2\hat{i} + \hat{j} - \hat{k}$, then $(a + b) \cdot (a - b)$ is equal to

(1) 0

(2) 12

(3) 6

(4) -6

14. The area of the triangle with vertices A(1, 1, 1), B(1, 2, 3) and C(2, 3, 1) is

(1) $\sqrt{21}$

(2) $\sqrt{14}/2$

(3) $\sqrt{21}/2$

(4) $21/2$

15. The points A(1, k, 3), B(2, 4, 5) and C(4, 8, 9) are collinear. Then k is equal to

(1) 4

(2) 2

(3) -2

(4) $8/3$

16. Statement I: $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$ for all vectors $\vec{a}$ and $\vec{b}$.
Statement II: $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$ for all vectors $\vec{a}$ and $\vec{b}$.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are false

17. Assertion A: $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$ for all non-zero vectors $\vec{a}$ and $\vec{b}$.
Reason R: $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) Both A and R are true but R is NOT the correct explanation of A

(3) A is true but R is false

(4) A is false but R is true

18. The vector of magnitude 9 in the direction of $2\hat{i} - \hat{j} + 2\hat{k}$ is

(1) $6\hat{i} - 3\hat{j} + 6\hat{k}$

(2) $18\hat{i} - 9\hat{j} + 18\hat{k}$

(3) $3\hat{i} - (3/2)\hat{j} + 3\hat{k}$

(4) $2\hat{i} - \hat{j} + 2\hat{k}$

19. If $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \hat{i} - \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} + 2\hat{k}$, then the projection of $\vec{a} \times \vec{b}$ on $\vec{c}$ is

(1) $1/3$

(2) $-1/3$

(3) -1

(4) $-\sqrt{3}/3$

20. If $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $\vec{a} \cdot \vec{b} = 1$, then $|(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b})|$ is equal to

(1) $\sqrt{3}$

(2) 4

(3) 2

(4) $2\sqrt{3}$

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

21. If $|\vec{a}| = 2$, $|\vec{b}| = 5$ and $|\vec{a} \times \vec{b}| = 8$, then $|\vec{a} \cdot \vec{b}|$ is equal to _____.

Answer: _____

22. If $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$, then $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2$ is equal to _____.

Answer: _____

23. If $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$, then $|\vec{a} \times \vec{b}|^2$ is equal to _____.

Answer: _____

24. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors, then $|2\vec{a} + 3\vec{b} + 6\vec{c}|$ is equal to _____.

Answer: _____

25. The area of the triangle with vertices (0, 0, 0), (1, 2, k) and (2, 1, 0) is $\sqrt{29}/2$. If $k > 0$, then k is equal to _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(3)	(1)	(4)	(2)	(1)	(3)	(4)	(1)	(2)	(3)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(4)	(2)	(1)	(3)	(2)	(3)	(4)	(1)	(2)	(4)

Q21	Q22	Q23	Q24	Q25
6	36	9	7	2

WORKED SOLUTIONS

Q1. Answer (3). $\vec{a} + \vec{b} = \hat{i} + 0\hat{j} + 5\hat{k}$, with magnitude $\sqrt{(1 + 25)} = \sqrt{26}$. The unit vector is $(\hat{i} + 5\hat{k})/\sqrt{26}$.

Q2. Answer (1). External division in the ratio m : n gives $(mq - np)/(m - n) = [2(-\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} - \hat{k})]/1 = -3\hat{i} + 0\hat{j} + 3\hat{k}$.

Q3. Answer (4). Parallel vectors have proportional components: $\lambda/2 = 6/3 = \mu/6 = 2$, so $\lambda = 4$ and $\mu = 12$. $\lambda + \mu = 16$.

Q4. Answer (2). $|\vec{a} + 2\vec{b}|^2 = |\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a}\cdot\vec{b} = 9 + 100 + 4 \times 3 \times 5 \times (-1/2) = 109 - 30 = 79$.

Q5. Answer (1). $\vec{c} = -(\vec{a} + \vec{b})$, so $49 = 9 + 25 + 2\vec{a}\cdot\vec{b}$ and $\vec{a}\cdot\vec{b} = 15/2$. $\cos\theta = (15/2)/(3 \times 5) = 1/2$, $\theta = \pi/3$.

Q6. Answer (3). The vector component is $(\vec{a}\cdot\vec{b}/|\vec{b}|^2)\,\vec{b} = (3/2)(\hat{i} + \hat{j})$.

Q7. Answer (4). The area is $|\vec{d}_1 \times \vec{d}_2|/2$. $\vec{d}_1 \times \vec{d}_2 = -2\hat{i} - 14\hat{j} - 10\hat{k}$, with magnitude $\sqrt{(4 + 196 + 100)} = 10\sqrt{3}$. Area $= 5\sqrt{3}$.

Q8. Answer (1). $(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{a} \times \vec{b} - \vec{a} \times \vec{c} - \vec{d} \times \vec{b} + \vec{d} \times \vec{c} = (\vec{a} \times \vec{b} - \vec{c} \times \vec{d}) - (\vec{a} \times \vec{c} - \vec{b} \times \vec{d})$, using $\vec{d} \times \vec{b} = -\vec{b} \times \vec{d}$ and $\vec{d} \times \vec{c} = -\vec{c} \times \vec{d}$. Both brackets vanish, so the result is the zero vector.

Q9. Answer (2). $|\vec{a} \times \vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = |\vec{a}|^2|\vec{b}|^2$. So $16|\vec{b}|^2 = 144$ and $|\vec{b}| = 3$.

Q10. Answer (3). $|\vec{a} + \vec{b} + \vec{c}|^2 = 3 + 2(\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}) = 0$, so the sum is $-3/2$.

Q11. Answer (4). Squaring: $2 + 2\cos\theta = 3(2 - 2\cos\theta)$, so $8\cos\theta = 4$, $\cos\theta = 1/2$ and $\theta = \pi/3$.

Q12. Answer (2). The displacement is $\vec{d} = 2\hat{i} + 3\hat{j} + \hat{k}$, and $W = \vec{F}\cdot\vec{d} = 4 + 3 + 1 = 8$ units.

Q13. Answer (1). $(\vec{a} + \vec{b})\cdot(\vec{a} - \vec{b}) = |\vec{a}|^2 - |\vec{b}|^2 = (1 + 1 + 4) - (4 + 1 + 1) = 0$.

Q14. Answer (3). $\vec{AB} = \hat{j} + 2\hat{k}$ and $\vec{AC} = \hat{i} + 2\hat{j}$. $\vec{AB} \times \vec{AC} = -4\hat{i} + 2\hat{j} - \hat{k}$, with magnitude $\sqrt{21}$. Area $= \sqrt{21}/2$.

Q15. Answer (2). $\vec{AB} = (1, 4 - k, 2)$ and $\vec{AC} = (3, 8 - k, 6)$ must be parallel: $3(4 - k) = 8 - k$, so $k = 2$.

Q16. Answer (3). Statement I is false: $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$ (for example $\hat{i} \times \hat{j} = \hat{k}$ but $\hat{j} \times \hat{i} = -\hat{k}$). Statement II is true: $\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$.

Q17. Answer (4). R is true. A is false: $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|$ only when $\sin\theta = 1$ (perpendicular vectors); for example $\vec{a} \times \vec{a} = 0$.

Q18. Answer (1). $|2\hat{i} - \hat{j} + 2\hat{k}| = 3$, so the vector is $9 \times (2\hat{i} - \hat{j} + 2\hat{k})/3 = 6\hat{i} - 3\hat{j} + 6\hat{k}$.

Q19. Answer (2). $\vec{a} \times \vec{b} = -\hat{i} + \hat{j} - \hat{k}$. Projection on $\vec{c} = (\vec{a} \times \vec{b})\cdot\vec{c}/|\vec{c}| = (-1 + 2 - 2)/3 = -1/3$.

Q20. Answer (4). $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = -\vec{a} \times \vec{b} + \vec{b} \times \vec{a} = -2(\vec{a} \times \vec{b})$. $|\vec{a} \times \vec{b}| = \sqrt{(|\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2)} = \sqrt{3}$, so the answer is $2\sqrt{3}$.

Q21. Answer 6. $(\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2 - |\vec{a} \times \vec{b}|^2 = 100 - 64 = 36$, so $|\vec{a}\cdot\vec{b}| = 6$.

Q22. Answer 36. $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2) = 2(9 + 9) = 36$.

Q23. Answer 9. $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, so $|\vec{a} \times \vec{b}|^2 = 4 + 4 + 1 = 9$.

Q24. Answer 7. The cross terms vanish: $|2a + 3b + 6c|^2 = 4 + 9 + 36 = 49$, so the magnitude is 7.

Q25. Answer 2. $(\hat{i} + 2\hat{j} + k\hat{k}) \times (2\hat{i} + \hat{j}) = -k\hat{i} + 2k\hat{j} - 3\hat{k}$, with magnitude $\sqrt{(5k^2 + 9)}$. Area $= \sqrt{(5k^2 + 9)}/2 = \sqrt{29}/2$ gives $k^2 = 4$, $k = 2$.

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: products were computed from the components in exact arithmetic, conditions were tested on explicitly constructed (and randomly rotated) vectors, and identities on random vectors.