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Gravitation — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · PHYSICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Take g = 10 m/s² unless stated otherwise. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Universal law of gravitation · Acceleration due to gravity · Variation of g with altitude and depth · Kepler's laws of planetary motion · Gravitational field · Gravitational potential · Gravitational potential energy · Escape velocity · Earth satellites · Energy of an orbiting satellite

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. The acceleration due to gravity at a height R/2 above the surface of the Earth (radius R) is equal to its value at a depth d below the surface. The value of d is

(1) R/4

(2) 5R/9

(3) R/2

(4) 4R/9

2. A planet has 1/9 of the mass of the Earth and 1/2 of its radius. A person who can jump 0.5 m high on the Earth can jump, with the same take-off speed, to a height on the planet of

(1) 0.5 m

(2) 2.25 m

(3) 1.125 m

(4) 0.22 m

3. If the Earth rotated about its axis so fast that bodies at the equator just felt weightless, the length of a day would be about (R = 6400 km, g = 10 m/s²)

(1) 84 minutes

(2) 24 hours

(3) 42 minutes

(4) 1 minute

4. Three particles, each of mass m, are at the vertices of an equilateral triangle of side a, and a particle of mass M is at its centroid. If one of the three particles is removed, the gravitational force on M becomes

(1) zero

(2) $3GmM/a^2$

(3) $\sqrt{3}\,GmM/a^2$

(4) GmM/a^2

5. For a uniform solid sphere of mass M and radius R, the ratio of the magnitude of its gravitational field at a distance R/2 from its centre to that at a distance 2R from its centre is

(1) 1 : 1

(2) 1 : 2

(3) 4 : 1

(4) 2 : 1

6. Imagine a narrow tunnel bored to the centre of a uniform planet of mass M and radius R. The least speed with which a body must be projected from the centre, up the tunnel, so that it escapes from the planet, is

(1) $\sqrt{(3GM/R)}$

(2) $\sqrt{(2GM/R)}$

(3) $\sqrt{(GM/R)}$

(4) $\sqrt{(5GM/2R)}$

7. A satellite moves round a planet of radius R in an elliptical orbit whose nearest and farthest distances from the centre of the planet are 2R and 6R. The ratio of its speed at the nearest point to that at the farthest point, and its period, are

(1) 9, and the same as that of a circular orbit of radius 4R

(2) 3, and the same as that of a circular orbit of radius 6R

(3) 3, and the same as that of a circular orbit of radius 4R

(4) $\sqrt{3}$, and the same as that of a circular orbit of radius 4R

8. Assertion A: The line joining a planet to the Sun sweeps out equal areas in equal intervals of time.
Reason R: The gravitational force of the Sun on a planet is a central force, so the angular momentum of the planet about the Sun is conserved.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is true but R is false

(3) A is false but R is true

(4) Both A and R are true and R is the correct explanation of A

9. A planet has twice the radius and twice the mean density of the Earth. The escape speed from its surface is

(1) $2\sqrt{2}$ times that from the Earth

(2) 2 times that from the Earth

(3) 4 times that from the Earth

(4) $\sqrt{2}$ times that from the Earth

10. A satellite of mass m moves in a circular orbit of radius 2R round the Earth (radius R; g at the surface). The least energy needed to shift it to a circular orbit of radius 3R is

(1) mgR/6

(2) mgR/4

(3) mgR/12

(4) mgR/3

11. The total mechanical energy of a satellite in a circular orbit is $-E$. Its kinetic energy, and the least additional energy that would make it escape from the Earth, are

(1) 2E and E

(2) E and 2E

(3) E and E

(4) E/2 and E

12. The Moon goes round the Earth in 27.3 days in an orbit of radius 3.84×10^5 km. Using Kepler's third law, the radius of the orbit of a satellite whose period is 1 day is about

(1) 3.6×10^4 km

(2) 1.4×10^4 km

(3) 9.6×10^4 km

(4) 4.2×10^4 km

13. The work needed to raise a body of mass m from a height R to a height 3R above the surface of the Earth (radius R; g at the surface) is

(1) mgR/4

(2) 2mgR

(3) mgR/2

(4) mgR/12

14. A body is projected vertically upward from the surface of the Earth (radius R) with half the escape speed. Ignoring air resistance, the greatest height it reaches above the surface is

(1) R/4

(2) R/2

(3) R/3

(4) R

15. A thin uniform ring has mass M and radius a. Along its axis, the magnitude of its gravitational field is greatest at a distance from its centre of

(1) a

(2) a$\sqrt{2}$

(3) a/2

(4) a/$\sqrt{2}$

16. A straight smooth tunnel is dug along a diameter of the Earth, taken as a uniform sphere (R = 6400 km, g = 10 m/s^2). A body dropped into it at one end reaches the other end after about

(1) 84 minutes

(2) 21 minutes

(3) 42 minutes

(4) 60 minutes

17. Two stars of masses m and 2m, a distance d apart, move in circles about their common centre of mass under their mutual gravitation alone. The period of their motion is

(1) $2\pi\sqrt{(d^3/(Gm))}$

(2) $2\pi\sqrt{(d^3/(2Gm))}$

(3) $2\pi\sqrt{(d^3/(3Gm))}$

(4) $2\pi\sqrt{(3d^3/(Gm))}$

18. Taking the radius of the Earth as 6400 km, the percentage decrease in the acceleration due to gravity at a height of 32 km above the surface, and at a depth of 32 km below it, are about

(1) 1% and 0.5%

(2) 0.5% and 1%

(3) 1% and 1%

(4) 0.5% and 0.5%

19. Consider the following statements.

(A) An astronaut in an orbiting satellite feels weightless because the Earth's gravity is zero there.

(B) An astronaut in an orbiting satellite feels weightless because he and the satellite fall freely with the same acceleration.

(C) The orbital speed of a satellite in a given circular orbit does not depend on its mass.

(D) The period of a satellite orbiting close to the Earth's surface depends on the mass of the satellite.

(E) A geostationary satellite must orbit in the Earth's equatorial plane.

Choose the correct answer from the options given below:

(1) (A), (C) and (E) only

(2) (B), (C) and (D) only

(3) (B) and (E) only

(4) (B), (C) and (E) only

20. A satellite orbits close to the surface of a planet with a period of 1.5 hours. The mean density of the planet is about ($G = 6.67 \times 10^{-11}$ N m^2 kg^{-2})

(1) 1.6×10^3 kg/m^3

(2) 4.8×10^3 kg/m^3

(3) 9.7×10^3 kg/m^3

(4) 2.4×10^3 kg/m^3

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

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21. The escape speed from the surface of the Earth is 11.2 km/s. The escape speed (in km/s) from the surface of a planet with 9 times the mass and 4 times the radius of the Earth is _____ (to the nearest integer).

Answer: _____

22. The height (in km) above the surface of the Earth at which the acceleration due to gravity is 64% of its value at the surface is _____ (radius of the Earth 6400 km).

Answer: _____

23. A satellite moves in a circular orbit at a height of 3R above the surface of the Earth (R = 6400 km, g = 10 m/s^2 at the surface). Its period (in hours) is _____ (to the nearest integer; take $\pi = 3.14$).

Answer: _____

24. The least energy (in MJ) needed to put a satellite of mass 100 kg, starting from rest on the surface of the Earth, into a circular orbit at a height equal to the Earth's radius is _____ (R = 6400 km, g = 10 m/s^2; ignore the Earth's rotation).

Answer: _____

25. Four particles, each of mass 1 kg, are at the corners of a square of side 1 m. The magnitude of the net gravitational force on any one of them is $x \times 10^{-2}$ G newton, where G is the gravitational constant in SI units. The value of x is _____ (to the nearest integer).

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(2)	(3)	(1)	(2)	(4)	(1)	(3)	(4)	(1)	(3)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(3)	(4)	(1)	(3)	(4)	(3)	(3)	(1)	(4)	(2)

Q21	Q22	Q23	Q24	Q25
17	1600	11	4800	191

WORKED SOLUTIONS

Q1. Answer (2). At height R/2: g' = g/(1 + 1/2)² = 4g/9. At depth d: g' = g(1 − d/R). So 1 − d/R = 4/9 and d = 5R/9. (The rule d = 2h holds only for small heights.)

Q2. Answer (3). g ∝ M/R², so g_{p}lanet = g × (1/9)/(1/2)² = 4g/9. With the same take-off speed the height is v²/(2g) ∝ 1/g: 0.5 × 9/4 = 1.125 m.

Q3. Answer (1). Weightless at the equator means gravity just supplies the centripetal force: ω²R = g, ω = √(10/6.4 × 10^{6}) = 1.25 × 10^{-3} rad/s. Day = 2π/ω ≈ 5030 s ≈ 84 minutes, the period of a satellite skimming the surface.

Q4. Answer (2). With all three present, the forces on M cancel by symmetry. Removing one leaves a net force equal and opposite to the force that particle exerted. The centroid is a/√3 from each vertex, so F = GmM/(a²/3) = 3GmM/a², directed away from the removed particle's position.

Q5. Answer (4). Inside, only the mass within r pulls: E = GM r/R³, so at R/2, E = GM/(2R²). Outside, E = GM/r², so at 2R, E = GM/(4R²). Ratio 2 : 1.

Q6. Answer (1). The potential at the centre of a uniform sphere is −3GM/(2R) (it is −GM/R at the surface). To reach infinity, ½v² = 3GM/(2R), so v = √(3GM/R), √(3/2) times the escape speed from the surface.

Q7. Answer (3). At the two ends of the major axis the velocity is perpendicular to the radius, so constant angular momentum gives v_{1}(2R) = v_{2}(6R): v_{1}/v_{2} = 3. By Kepler's third law the period depends only on the semi-major axis, (2R + 6R)/2 = 4R.

Q8. Answer (4). The areal velocity is dA/dt = L/(2m). The force on the planet always points towards the Sun, so its torque about the Sun is zero and L is constant; hence dA/dt is constant (Kepler's second law). Both are true and R explains A.

Q9. Answer (1). v_{e} = √(2GM/R) with M = (4/3)πR³ρ, so v_{e} = R√(8πGρ/3) ∝ R√ρ. Doubling R and ρ multiplies v_{e} by 2√2.

Q10. Answer (3). Total energy in a circular orbit of radius r is −GMm/(2r) = −mgR²/(2r). From 2R to 3R: ΔE = mgR²(1/(4R) − 1/(6R)) = mgR/12. Using potential energy alone (−GMm/r) gives mgR/6, which ignores the loss of kinetic energy.

Q11. Answer (3). In a circular orbit, K = GMm/(2r), U = −GMm/r and the total is −GMm/(2r) = −E. So K = E, and raising the total energy to zero needs E more.

Q12. Answer (4). r ∝ T^(2/3): r = 3.84 × 10^{5} × (1/27.3)^(2/3) = 3.84 × 10^{5}/9.07 ≈ 4.2 × 10^{4} km, the radius of the geostationary orbit (its height above the surface is about 3.6 × 10^{4} km).

Q13. Answer (1). U = −GMm/r = −mgR²/r. From r = 2R to r = 4R: W = mgR²(1/(2R) − 1/(4R)) = mgR/4. Using mgh = 2mgR is wrong because g falls off with height.

Q14. Answer (3). v² = v_{e}²/4 = GM/(2R). Energy: ½m GM/(2R) − GMm/R = −GMm/r, so 1/r = (1/R)(1 − 1/4) and r = 4R/3: a height of R/3. (The constant-g formula v²/2g = R/4 underestimates it.)

Q15. Answer (4). On the axis at distance x, the components along the axis add: $E = GMx/(a^2 + x^2)^{(3/2)}$. $dE/dx = 0$ when $(a^2 + x^2) = 3x^2$, i.e. $x = a/\sqrt{2}$. The field is zero at the centre and falls off far away.

Q16. Answer (3). Inside, $g(r) = g\, r/R$ towards the centre, so the body moves in simple harmonic motion with $\omega = \sqrt{(g/R)}$ and period $2\pi\sqrt{(R/g)} \approx 84$ min. End to end is half a period, about 42 minutes.

Q17. Answer (3). The star of mass m moves in a circle of radius 2d/3 about the centre of mass: $G(2m)m/d^2 = m\omega^2(2d/3)$, so $\omega^2 = 3Gm/d^3 = G(m + 2m)/d^3$ and $T = 2\pi\sqrt{(d^3/(3Gm))}$. Taking the heavier star as fixed gives $2\pi\sqrt{(d^3/(2Gm))}$.

Q18. Answer (1). At a small height h, $\Delta g/g \approx 2h/R = 64/6400 = 1\%$. At depth d, $\Delta g/g = d/R = 32/6400 = 0.5\%$.

Q19. Answer (4). (A) is wrong: gravity at a few hundred km is nearly as strong as at the surface; it is what keeps the satellite in orbit. (B) is right: both are in free fall, so no contact force acts. (C) is right: $v = \sqrt{(GM/r)}$. (D) is wrong: $T = 2\pi\sqrt{(r^3/GM)}$. (E) is right: to stay above one point the orbit must be centred on the Earth's axis and lie in the equatorial plane.

Q20. Answer (2). For a close orbit, $GM/R^2 = (2\pi/T)^2R$, and $M = (4/3)\pi R^3\rho$. So $\rho = 3\pi/(GT^2) = 3\pi/(6.67 \times 10^{-11} \times 5400^2) \approx 4.8 \times 10^3$ kg/m³, whatever the planet's size.

Q21. Answer 17. $v_e = \sqrt{(2GM/R)} \propto \sqrt{(M/R)} = \sqrt{(9/4)} = 1.5$, so $v_e = 1.5 \times 11.2 = 16.8$ km/s, i.e. 17.

Q22. Answer 1600. $(R/(R + h))^2 = 0.64$, so $R/(R + h) = 0.8$ and $R + h = 1.25R$: $h = R/4 = 1600$ km.

Q23. Answer 11. $r = 4R$ and $T = 2\pi\sqrt{(r^3/(gR^2))} = 2\pi\sqrt{(64R/g)} = 16\pi\sqrt{(R/g)} = 16 \times 3.14 \times 800$ s $\approx 40\,200$ s ≈ 11.2 hours, i.e. 11.

Q24. Answer 4800. On the surface $E = -GMm/R = -mgR$. In the orbit ($r = 2R$), $E = -GMm/(4R) = -mgR/4$. The energy needed is $3mgR/4 = 0.75 \times 100 \times 10 \times 6.4 \times 10^6 = 4.8 \times 10^9$ J = 4800 MJ.

Q25. Answer 191. The two neighbours each pull with G (at 90° to each other), giving $G\sqrt{2}$ along the diagonal; the opposite corner pulls with G/2 along the same diagonal. Total $= G(\sqrt{2} + 1/2) \approx 1.914G = 191 \times 10^{-2}$ G.

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