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Rotational Motion — JEE (Main) Pattern Topic Test

FREE SAMPLE TEST · PHYSICS · PATTERN OF JEE (MAIN)

DURATION	MAXIMUM MARKS	QUESTIONS	NAME / ROLL NO.
60 minutes	100	25 (20 + 5)	

GENERAL INSTRUCTIONS

1. This question paper has two sections, A and B. All 25 questions are compulsory.
2. Section A has TWENTY (20) multiple-choice questions. Each has FOUR options, (1), (2), (3) and (4), of which ONLY ONE is correct.
3. Section B has FIVE (05) questions whose answer is a numerical value. Enter the value rounded off to the nearest integer.
4. Marking scheme for both sections: +4 for a correct answer, −1 for an incorrect answer, 0 if the question is not attempted.
5. Take g = 10 m/s² unless stated otherwise. Use of a calculator, log tables or any other aid is not allowed.
6. The answer key and worked solutions are at the end of this booklet. Attempt the paper in 60 minutes before you look at them.

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Topics covered: Centre of mass of a two-particle system · Centre of mass of a rigid body · Moment of inertia · Radius of gyration · Theorem of parallel axes · Theorem of perpendicular axes · Torque · Angular momentum · Conservation of angular momentum · Equilibrium of rigid bodies · Rotational kinematics · Equations of rotational motion · Comparison of linear and rotational motion

SECTION A (Q1–20) · Multiple choice · +4 / 0 / −1

Each question has FOUR options. ONLY ONE of them is correct.

1. From a uniform square plate of side a, a square piece of side a/2 is cut off at one corner, O. Taking O as the origin and the edges of the original plate through O as the x- and y-axes, the centre of mass of the remaining plate is at

(1) (7a/12, 7a/12)

(2) (5a/12, 5a/12)

(3) (2a/3, 2a/3)

(4) (7a/16, 7a/16)

2. The linear mass density of a rod of length L increases from one end, x = 0, as $\lambda = \lambda_0(1 + x/L)$. The distance of its centre of mass from the end x = 0 is

(1) L/2

(2) 2L/3

(3) 5L/9

(4) 4L/7

3. Particles of masses 1 kg and 3 kg are held at rest at x = 0 and x = 4 m on a smooth horizontal surface, and are then released. They move towards each other under their mutual attraction alone. They meet at

(1) x = 2 m

(2) x = 3 m

(3) x = 1 m

(4) x = 4/3 m

4. Match List-I (uniform bodies of radius R) with List-II (distance of the centre of mass from the centre of the full circle or sphere).
List-I: (A) Semicircular wire (B) Semicircular plate (C) Solid hemisphere (D) Hemispherical shell
List-II: (I) 3R/8 (II) R/2 (III) 2R/π (IV) 4R/(3π)
Choose the correct answer from the options given below:

(1) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(4) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

5. The moment of inertia of a uniform rod of mass M and length L about an axis perpendicular to it, passing through a point at a distance L/4 from one end, is

(1) $ML^2/48$

(2) $7ML^2/48$

(3) $ML^2/16$

(4) $13ML^2/48$

6. For a uniform disc, the ratio of its moment of inertia about a tangent lying in its plane to its moment of inertia about a tangent perpendicular to its plane is

(1) 6 : 5

(2) 1 : 2

(3) 5 : 6

(4) 3 : 2

7. The radius of gyration of a uniform square plate of side a about one of its diagonals is

(1) a/2

(2) a/√6

(3) a/√3

(4) a/(2√3)

8. Three particles, each of mass m, are at the vertices of an equilateral triangle of side a. Their moment of inertia about an axis through one vertex perpendicular to the plane of the triangle, and about an axis along one side, are respectively

(1) $2ma^2$ and $3ma^2/4$

(2) $2ma^2$ and $3ma^2/2$

(3) ma^2 and $3ma^2/4$

(4) $3ma^2/2$ and $ma^2/4$

9. A force $\vec{F} = (2\hat{i} - 3\hat{j} + \hat{k})$ N acts at the point with position vector $\vec{r} = (\hat{i} + \hat{j} + \hat{k})$ m. The torque of F about the origin is

(1) $(4\hat{i} + \hat{j} - 5\hat{k})$ N m

(2) $(-4\hat{i} - \hat{j} + 5\hat{k})$ N m

(3) $(4\hat{i} - \hat{j} - 5\hat{k})$ N m

(4) $(2\hat{i} + \hat{j} - 5\hat{k})$ N m

10. A light string wound round a uniform disc of mass 4 kg and radius 0.2 m, free to turn about its fixed horizontal axis, carries a block of mass 2 kg. The block is released from rest. Its acceleration and the tension in the string are (g = 10 m/s²)

(1) 10/3 m/s² and 40/3 N

(2) 5 m/s² and 10 N

(3) 5 m/s² and 20 N

(4) 20/3 m/s² and 20/3 N

11. Blocks of masses 3 kg and 5 kg hang from the ends of a light string that passes over a pulley, a uniform disc of mass 4 kg, without slipping. The pulley turns freely about its axle. The acceleration of the blocks and the tensions in the string on the 3 kg and the 5 kg side are (g = 10 m/s²)

(1) 2.5 m/s², 37.5 N and 37.5 N

(2) 2 m/s², 40 N and 36 N

(3) 2 m/s², 36 N and 40 N

(4) 2 m/s², 36 N and 36 N

12. A uniform disc rotates freely about its fixed vertical axis with angular speed ω_0. A thin ring of the same mass and radius, not rotating, is placed gently and coaxially on it, and the two soon rotate together. The common angular speed, and the fraction of the initial kinetic energy that is lost, are

(1) $\omega_0/2$ and 1/2

(2) $\omega_0/3$ and 1/3

(3) $\omega_0/3$ and 2/3

(4) $2\omega_0/3$ and 1/3

13. A ball of mass 0.1 kg is projected from the origin with a speed of 20 m/s at 45° to the horizontal. The magnitude of its angular momentum about the origin when it is at the highest point of its path is ($g = 10\ m/s^2$)

(1) $10\sqrt{2}$ kg m² s⁻¹

(2) $20\sqrt{2}$ kg m² s⁻¹

(3) $5\sqrt{2}$ kg m² s⁻¹

(4) 10 kg m² s⁻¹

14. A uniform ladder of length 5 m and mass 20 kg rests with its upper end against a smooth vertical wall, 4 m above the rough horizontal floor. The least coefficient of friction between the ladder and the floor for which the ladder does not slip is

(1) 0.75

(2) 0.5

(3) 0.25

(4) 0.375

15. A uniform beam of length 2 m and mass 4 kg rests on two knife-edges, at 0.4 m and 1.6 m from its left end. A load of 6 kg is placed on the beam 1.8 m from its left end. The reactions at the left and right knife-edges are ($g = 10\ m/s^2$)

(1) 10 N and 90 N

(2) 40 N and 60 N

(3) 50 N and 50 N

(4) 90 N and 10 N

16. A wheel turning at 1200 rpm is brought to rest with a uniform angular retardation in 20 s. The number of revolutions it makes in the last 5 s before it stops is

(1) 50

(2) 25

(3) 6.25

(4) 12.5

17. A uniform rod of mass M and length L is hinged at one end so that it can turn freely in a vertical plane. It is held horizontal and then released. The force exerted on the rod by the hinge just after release is

(1) Mg/2

(2) Mg/4

(3) Mg

(4) 3Mg/4

18. A uniform rod of mass m and length L, hinged freely at its lower end, is released from rest in the vertical (upright) position and falls. When it passes the horizontal position, its angular speed and the magnitude of the force exerted by the hinge are

(1) $\sqrt{(3g/L)}$ and $(\sqrt{37}/4)mg$

(2) $\sqrt{(3g/L)}$ and $3mg/2$

(3) $\sqrt{(3g/2L)}$ and $mg/4$

(4) $\sqrt{(6g/L)}$ and $(\sqrt{37}/4)mg$

19. Statement I: The angular momentum of a particle moving in a straight line with constant velocity is the same at all times, about any fixed point.
Statement II: The torque about a point of a force whose line of action always passes through that point is zero.
In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are false

(2) Statement I is true but Statement II is false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

20. A solid sphere of mass 5 kg and radius 0.2 m, free to turn about a fixed axis through its centre, is spun up from rest to 10 rad/s by a constant torque of 0.4 N m. The work done by the torque and the time taken are

(1) 4 J and 2 s

(2) 10 J and 5 s

(3) 4 J and 5 s

(4) 2 J and 1 s

SECTION B (Q21–25) · Numerical value · +4 / 0 / −1

The answer to each question is a NUMERICAL VALUE. Enter it rounded off to the nearest integer.

21. A uniform disc has radius 30 cm. A circular hole of radius 10 cm is cut in it, the centre of the hole being 15 cm from the centre of the disc. The centre of mass of the remaining disc is displaced from the centre of the disc by a distance (in mm) of _____ (to the nearest integer).

Answer: _____

22. A uniform solid cylinder has mass 2 kg, radius 0.1 m and length 0.6 m. Its moment of inertia about an axis through its centre, perpendicular to its length, is $x \times 10^{-3}$ kg m². The value of x is _____.

Answer: _____

23. A horizontal platform in the form of a uniform disc of mass 120 kg and radius 2 m turns freely about a vertical axis through its centre at 10 rpm. A child of mass 30 kg stands at its edge. The child walks slowly to the centre. The new rate of rotation (in rpm) is _____.

Answer: _____

24. A uniform rod of length 1.2 m is hinged at one end so that it can turn freely in a vertical plane. It is released from rest in the horizontal position. The speed (in m/s) of its free end as it passes the vertical position is _____ ($g = 10$ m/s²).

Answer: _____

25. A uniform rod of mass 4 kg and length 1 m lies on a smooth horizontal table and is pivoted at its centre so that it can turn about a vertical axis. A horizontal force of 8 N is applied at one end, perpendicular to the rod. The angular acceleration (in rad/s²) of the rod is _____.

Answer: _____

END OF THE QUESTION PAPER

ANSWER KEY

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
(1)	(3)	(2)	(4)	(2)	(3)	(4)	(1)	(1)	(2)

Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20
(3)	(3)	(1)	(4)	(1)	(4)	(2)	(1)	(4)	(1)

Q21	Q22	Q23	Q24	Q25
19	65	15	6	12

WORKED SOLUTIONS

Q1. Answer (1). Treat the plate as the full square (mass 4m, centre (a/2, a/2)) minus the cut piece (mass m, centre (a/4, a/4)). $x_C m = (4m \times a/2 - m \times a/4)/(3m) = (2a - a/4)/3 = 7a/12$, and the same for y. The centre of mass moves away from the cut corner.

Q2. Answer (3). $x_C m = \int_0^L x\lambda\, dx / \int_0^L \lambda\, dx = \lambda_0(L^2/2 + L^2/3)/(\lambda_0(L + L/2)) = (5L^2/6)/(3L/2) = 5L/9$, a little beyond the midpoint, towards the heavier end.

Q3. Answer (2). Only internal forces act, so the centre of mass, initially at rest at $(1 \times 0 + 3 \times 4)/4 = 3$ m, stays there. The particles meet at the centre of mass, x = 3 m (the 1 kg particle moves three times as far).

Q4. Answer (4). Wire: $\bar{y} = \int R \sin\theta\, R\, d\theta/(\pi R) = 2R/\pi$. Plate: $\bar{y} = \iint r \sin\theta\, r\, dr\, d\theta/(\pi R^2/2) = 4R/(3\pi)$. Solid hemisphere: slices of radius $\sqrt{(R^2 - y^2)}$: $\bar{y} = \int y(R^2 - y^2)dy / \int (R^2 - y^2)dy = (R^4/4)/(2R^3/3) = 3R/8$. Shell: equal heights carry equal area, so $\bar{y} = R/2$.

Q5. Answer (2). About the centre $I = ML^2/12$; the point is L/4 from the centre, so $I = ML^2/12 + M(L/4)^2 = ML^2(4 + 3)/48 = 7ML^2/48$.

Q6. Answer (3). About its axis $I = MR^2/2$, so about a diameter $MR^2/4$ (perpendicular axes). Tangent in the plane: $MR^2/4 + MR^2 = 5MR^2/4$. Tangent perpendicular to the plane (parallel to the axis): $MR^2/2 + MR^2 = 3MR^2/2$. Ratio (5/4) : (3/2) = 5 : 6.

Q7. Answer (4). About the two axes through the centre parallel to the sides, $I = Ma^2/12$ each, so about the perpendicular axis $Ma^2/6$. The two diagonals are also perpendicular axes in the plane and are equivalent, so each has $I = Ma^2/12$. $k = \sqrt{(I/M)} = a/\sqrt{12} = a/(2\sqrt{3})$.

Q8. Answer (1). Axis through a vertex, perpendicular to the plane: the other two masses are at distance a each, $I = 2ma^2$. Axis along a side: two masses lie on it; the third is at the height $a\sqrt{3}/2$, so $I = m(3a^2/4) = 3ma^2/4$.

Q9. Answer (1). $\tau = r \times F = \hat{i}[(1)(1) - (1)(-3)] - \hat{j}[(1)(1) - (1)(2)] + \hat{k}[(1)(-3) - (1)(2)] = 4\hat{i} + \hat{j} - 5\hat{k}$ N m. F × r has the opposite sign.

Q10. Answer (2). Block: $20 - T = 2a$. Disc: $TR = (\tfrac{1}{2}MR^2)(a/R)$, so $T = \tfrac{1}{2}Ma = 2a$. Then $20 = 4a$: $a = 5$ m/s², $T = 10$ N (the disc turns with $\alpha = a/R = 25$ rad/s²). Treating the disc as a ring ($T = Ma$) gives 10/3 m/s².

Q11. Answer (3). $5g - T_2 = 5a$, $T_1 - 3g = 3a$ and $(T_2 - T_1)R = (\tfrac{1}{2} \times 4R^2)(a/R)$, i.e. $T_2 - T_1 = 2a$. Adding: $2g = (5 + 3 + 2)a$, so $a = 2$ m/s², $T_1 = 3 \times 12 = 36$ N and $T_2 = 5 \times 8 = 40$ N. The tensions differ because the pulley needs a net torque; with a light pulley $a = 2.5$ m/s² and both are 37.5 N.

Q12. Answer (3). $I_{disc} = MR^2/2 = I$ and $I_{ring} = MR^2 = 2I$. Angular momentum: $I\omega_0 = 3I\omega$, so $\omega = \omega_0/3$. Kinetic energy: $\tfrac{1}{2}(3I)(\omega_0/3)^2 = (1/3) \times \tfrac{1}{2}I\omega_0^2$, so 2/3 of it is lost (to friction between the two).

Q13. Answer (1). At the top the velocity is horizontal, $u \cos 45° = 10\sqrt{2}$ m/s, and the height is $H = u^2 \sin^2 45°/(2g) = 10$ m. $L = mvH$ (the lever arm of a horizontal momentum is the height) $= 0.1 \times 10\sqrt{2} \times 10 = 10\sqrt{2}$ kg m² s⁻¹ ≈ 14.1.

Q14. Answer (4). The foot is 3 m from the wall. Torques about the foot: N_{w}all × 4 = 200 × 1.5, so N_{w}all = 75 N. Horizontal balance: friction = 75 N; vertical balance: N_{f}loor = 200 N. So $\mu \geq 75/200 = 0.375$.

Q15. Answer (1). Torques about the left knife-edge (0.4 m): $R_2 \times 1.2 = 40 \times (1.0 - 0.4) + 60 \times (1.8 - 0.4) = 24 + 84 = 108$, so $R_2 = 90$ N. Then $R_1 = 100 - 90 = 10$ N. (A load beyond the right support nearly lifts the left end.)

Q16. Answer (4). $\omega_0 = 20$ rev/s and $\alpha = 1$ rev/s². Running the motion backwards from rest, the angle turned in the last t seconds is $\tfrac{1}{2}\alpha t^2 = \tfrac{1}{2} \times 1 \times 25 = 12.5$ rev (out of 200 rev in all).

Q17. Answer (2). About the hinge, $Mg(L/2) = (ML^2/3)\alpha$, so $\alpha = 3g/(2L)$ and the centre of mass accelerates downward at $\alpha(L/2) = 3g/4$. There is no centripetal part yet ($\omega = 0$). So $Mg - N = M(3g/4)$, $N = Mg/4$, upward.

Q18. Answer (1). Energy: $mg(L/2) = \tfrac{1}{2}(mL^2/3)\omega^2$, so $\omega = \sqrt{(3g/L)}$. In the horizontal position, $\alpha = 3g/(2L)$ as before. The centre of mass has centripetal acceleration $\omega^2 L/2 = 3g/2$ (towards the hinge) and tangential acceleration $\alpha L/2 = 3g/4$ (downward). Hinge force: $3mg/2$ horizontally and $mg - 3mg/4 = mg/4$ vertically, magnitude $mg\sqrt{(9/4 + 1/16)} = (\sqrt{37}/4)mg$.

Q19. Answer (4). Statement I: $L = m(r \times v) = mvd$, where d is the perpendicular distance of the line from the point, which does not change; the direction does not change either. True. Statement II: $\tau = r \times F = 0$ when F is along r. True (this is why angular momentum is conserved under central forces).

Q20. Answer (1). $I = (2/5)MR^2 = 0.4 \times 5 \times 0.04 = 0.08$ kg m². Work $= \tfrac{1}{2}I\omega^2 = \tfrac{1}{2} \times 0.08 \times 100 = 4$ J. Time: $\tau = I\alpha$, so $t = I\omega/\tau = 0.8/0.4 = 2$ s (the rotational analogue of $F\,t = mv$).

Q21. Answer 19. Masses are proportional to areas: the hole is 1/9 of the disc. Shift $= (1/9 \times 15)/(1 - 1/9) = 15/8 = 1.875$ cm ≈ 18.75 mm, away from the hole: 19 mm.

Q22. Answer 65. Slicing the cylinder into discs and using the perpendicular- and parallel-axes theorems for each, $I = M(R^2/4 + L^2/12) = 2(0.01/4 + 0.36/12) = 2(0.0025 + 0.03) = 0.065$ kg m² $= 65 \times 10^{-3}$ kg m².

Q23. Answer 15. $I_1 = \tfrac{1}{2} \times 120 \times 4 + 30 \times 4 = 240 + 120 = 360$ kg m²; $I_2 = 240$ kg m² (child at the axis). Angular momentum: $360 \times 10 = 240 \times n$, so $n = 15$ rpm.

Q24. Answer 6. Its centre of mass falls L/2: $mg(L/2) = \tfrac{1}{2}(mL^2/3)\omega^2$, so $\omega = \sqrt{(3g/L)} = \sqrt{25} = 5$ rad/s and the free end moves at $\omega L = 6$ m/s ($= \sqrt{(3gL)}$).

Q25. Answer 12. $\tau = 8 \times 0.5 = 4$ N m and $I = ML^2/12 = 1/3$ kg m², so $\alpha = \tau/I = 12$ rad/s².

All questions are original and © Master Testing Agency. Free to download and use for personal practice; not for resale. Every answer in this paper was checked independently by computer: centres of mass and moments of inertia by numerical integration over each body, torques and angular momenta from r × F and r × p, and rigid-body motions by step-by-step integration of the equations of rotation.